Static Electricity

In the beginning.

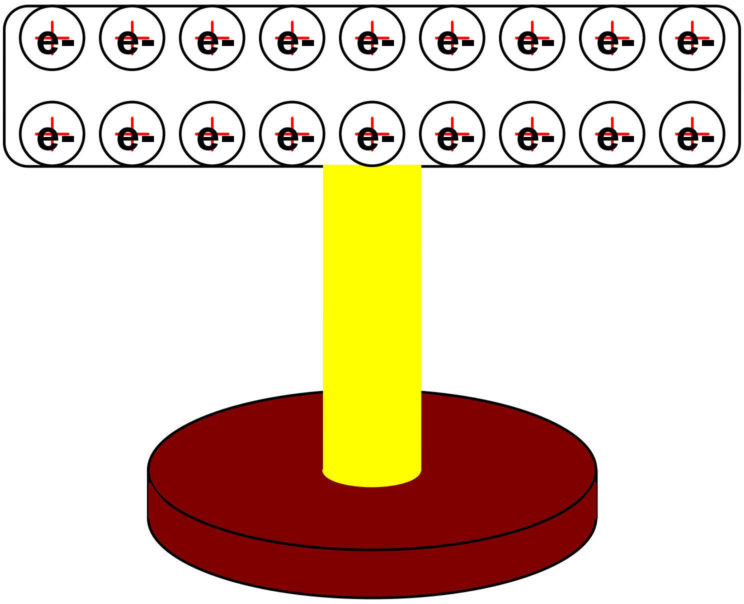
The ancient Greeks knew that when amber (fossilised tree resin) is rubbed with fur it will temporarily attract other small objects to it.
Scientists and philosophers through history have investigated this effect, both with amber and other materials. (Ancient Greek word for Amber - ηλεκτρων, elektron)
Amber is a very good insulator and we now know that when amber is rubbed with fur, wool, silk etc that electrons are transferred from the amber to the rubbing material.
This leaves the amber with a net loss of electrons, i.e. a positive charge.
Since the amber is an insulator, electrons cannot move and so this lack of electrons remains until free electrons from the air are able to replace those electrons that were rubbed off and the amber becomes uncharged again.

Early experiments with insulators determined that

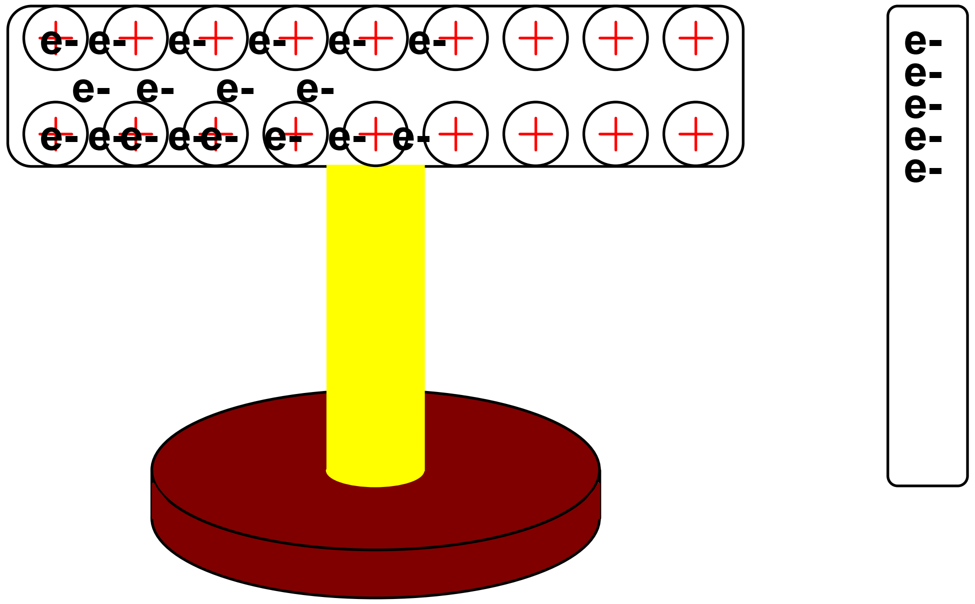
like charge repel each other and opposite charge attract.

Insulated conductors.

Consider a metal bar on an insulating stand.



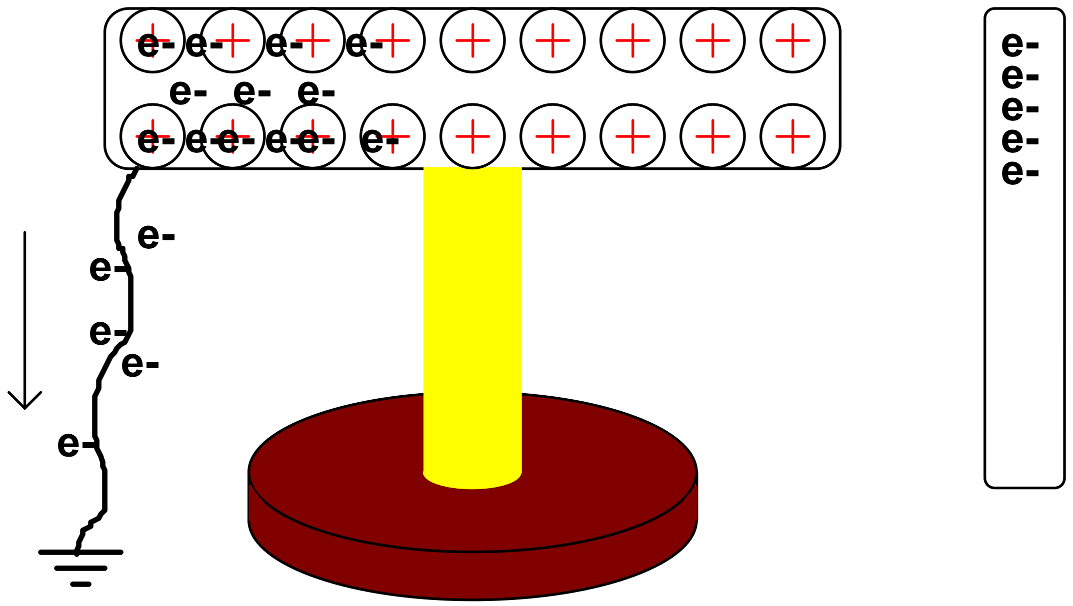
When uncharged, there are equal numbers of free electrons and metal ions.
Consider when a negatively charged insulator is brought near to the metal bar.



The electrons are repelled by the negatively charged insulator and move away from the charged insulator. There is now an attractive force between the metal bar and charged insulator.
The metal bar is still uncharged.

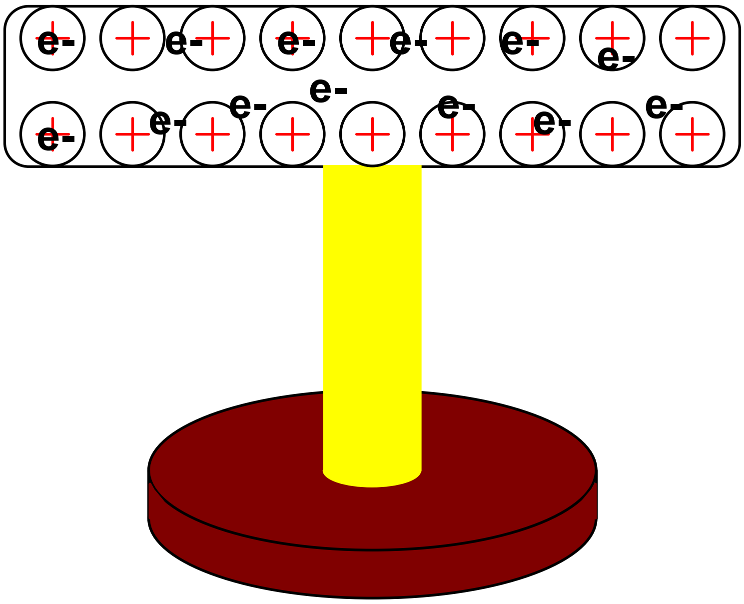
The earth is considered to be electrically uncharged and to have an electrical potential of 0V. Since the earth is so large, the addition of loss of a few electrons will not alter this situation.

Consider what happens if the metal bar is now connected to Earth.



Some of the displaced electrons will now be repelled along the wire to earth, so leaving the metal bar with a lack of electrons, i.e. a positive charge.

When the earth wire and charged insulator are removed, the electrons in the metal bar re-distribute themselves and the metal bar is left positively charged.



Charles Augustin de Coulomb

In the 18th century, Coulomb made measurements on the force between two charged materials and found that:

the force $\propto$ product of the two charges
the force $\propto$ 1./ distance between the charges squared
the force was directed between the two charges.

$$F \propto \frac{Q_1 Q_2}{d^2}$$

Although Coulomb's Law worked well, it worried scientists that an effect appeared to be happening at a distance, with nothing happening in between.

Electric Fields

As a result of further experiments and to overcome this problem the concept of an Electric Field is used.
An Electric field is the space around any charged object where another charged object will experience a force.
The electric field strength around a point charge (Q_1) is given the symbol E where

$$E \propto \frac{Q_1}{d^2}$$

d is the distance away from the charge.

The force acting on a small point charge (Q_2) in this electric field is then

$$F = Q_2 E$$

The units of E are Newtons/Coulomb, N/C (and also Volts/metre, V/m)
(Compare gravitational field strength, g, N/kg)

In SI units the constant of proportionality is Coulomb's Law is

$$\frac{1}{4\pi\varepsilon_0}$$

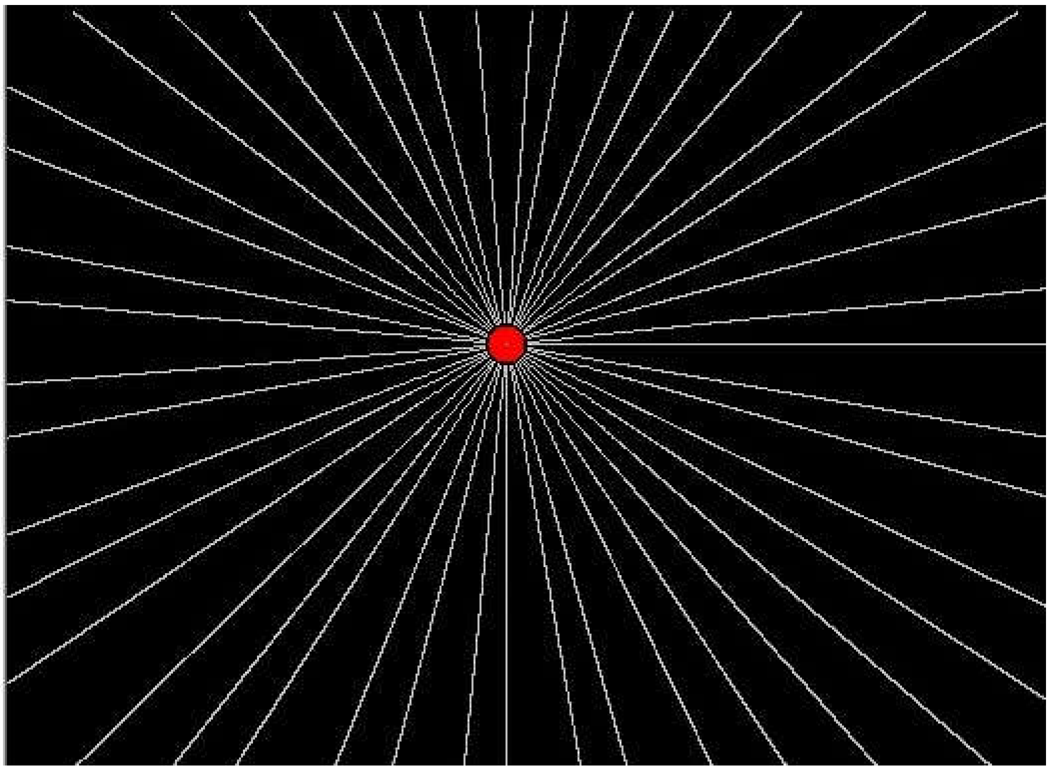
where ε_0 is the permittivity of free space = $8.85\text{x}10^{-12}\text{Fm}^{-1}$

$$\Rightarrow F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} \quad \text{and} \quad \Rightarrow E = \frac{Q_1}{4\pi\varepsilon_0 d^2}$$

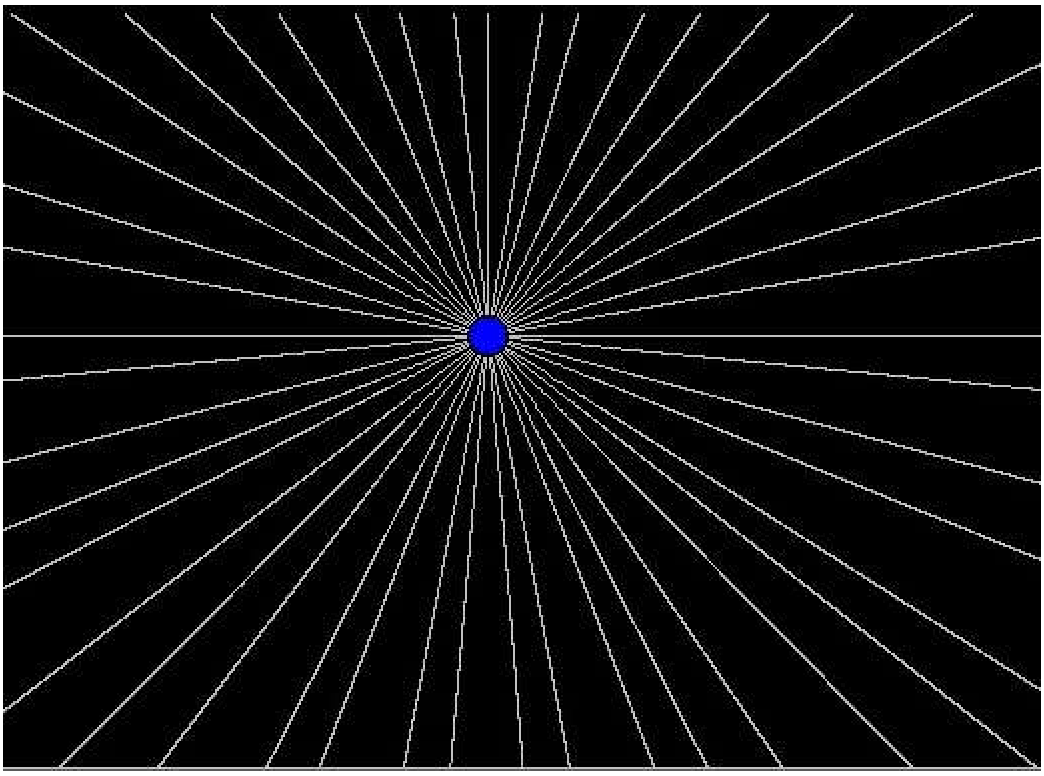
E is a vector quantity with direction given by the force acting on a small positive test charge.
NOTE: $4\pi d^2$ is the surface area of a sphere of radius d centred on charge Q_1.

Representing Electric Fields

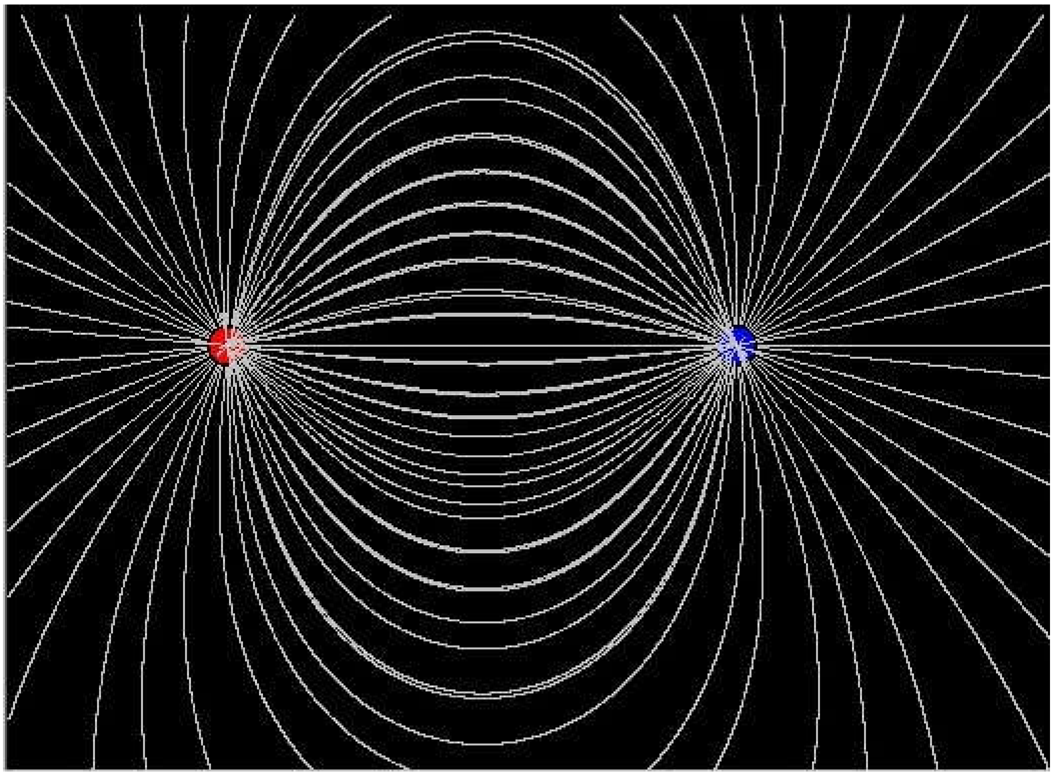
Electric fields can be represented by field lines, as with magnetic fields.
The direction of a field line at any point is given by the direction of the force acting on a small positive test charge.
The strength of the electric field is represented by the number of field lines per unit cross-sectional area. So the closer they are together the stronger the field.
If the field lines are parallel and equally spaced, then the field is uniform.
Field lines cut surfaces of equal potential at right angles.



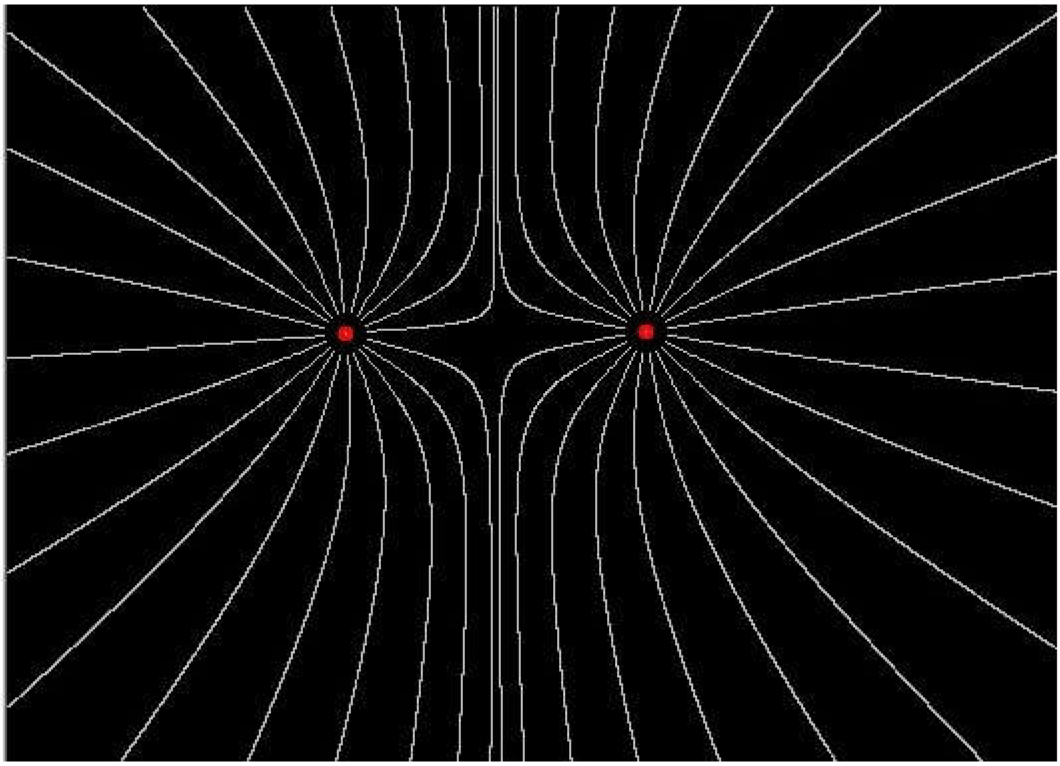
Positive point charge giving a radial field.
The direction of the Electric field strength is away from the point charge.



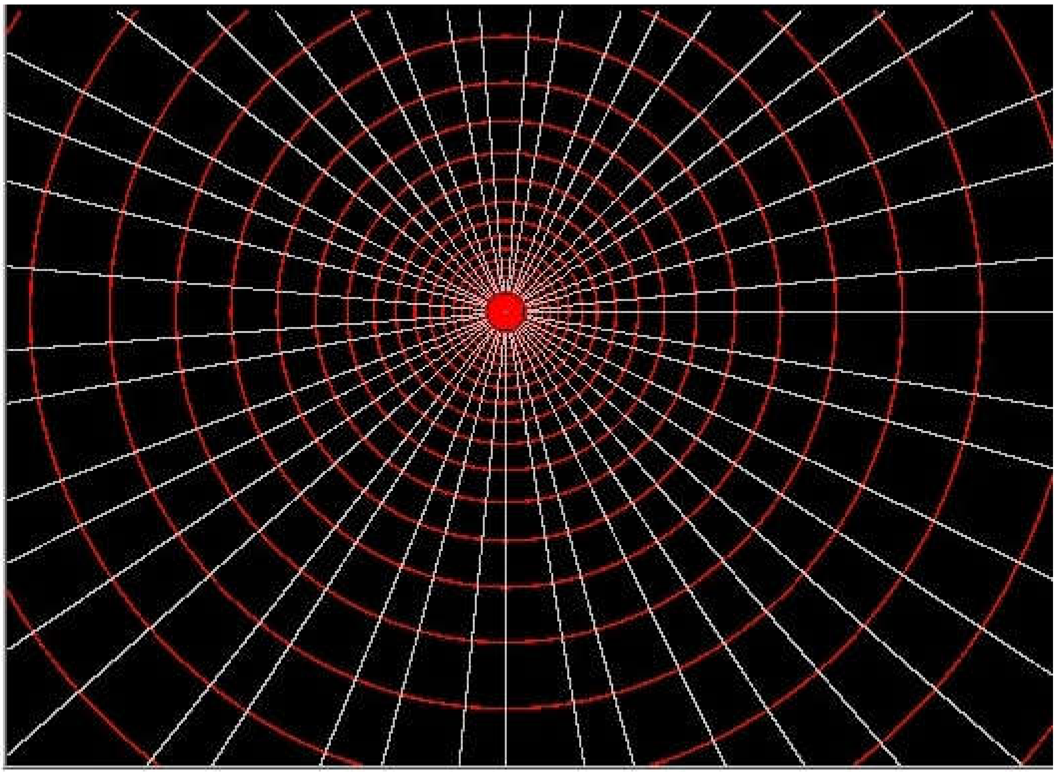
Negative point charge giving a radial field.
The direction of the Electric field strength is towards the point charge.



Positive and negative point charges.
Note that the field lines start on the positive charge and end on the negative charge.



Two positive point charges.
Note that the field lines start on the positive charge and end at infinity!



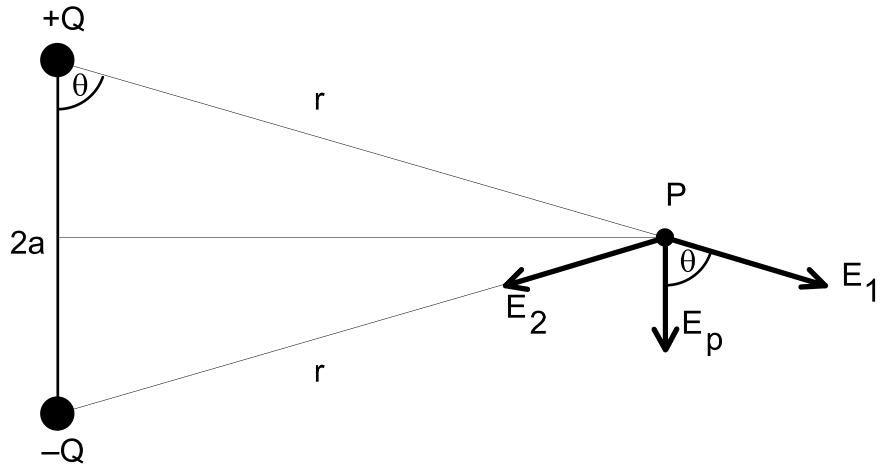
Positive point charge giving a radial field.
The red circles (spheres) indicate surfaces of equal electric potential energy.
Note that the field lines intersect the equal potential surfaces at right angles.

Calculating Electric Field strength due to several point charges.

Electric field strength is a vector quantity and so to calculate the electric field strength due to several point charges, the individual electric field strengths are added vectorially.

$$\overline{E} = \sum_{q=1}^{q=n} \overline{E_q}$$

E.g.
Consider a point positive and negative charge separated by a distance of 2a
This arrangement is called an electric dipole.



$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \bullet \frac{Q}{r^2}$$

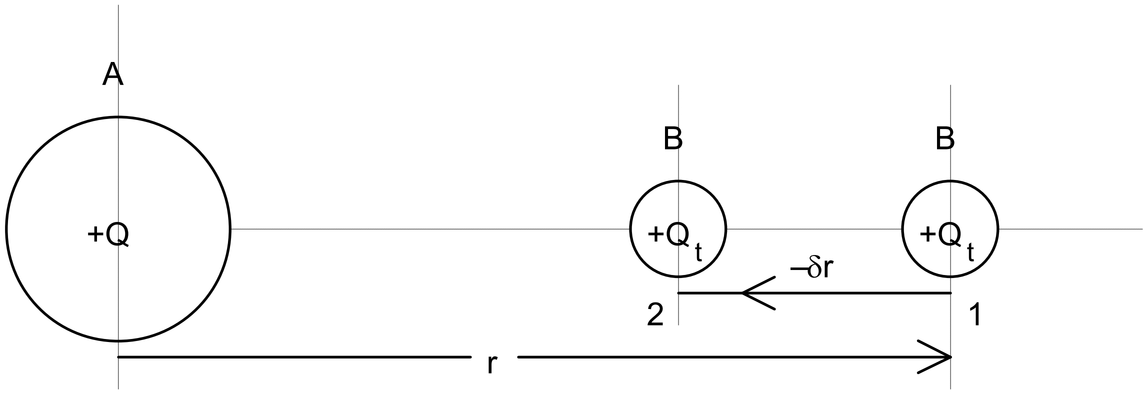
$$E_p = 2\,E_1 \cos\theta$$

$$\Rightarrow E_p = \frac{1}{4\pi\varepsilon_0} \bullet \frac{2Q}{r^2} \bullet \cos\theta = \frac{1}{4\pi\varepsilon_0} \bullet \frac{2Q}{r^2} \bullet \frac{a}{r}$$

$\Rightarrow E_p = \frac{1}{4\pi\varepsilon_0} \bullet \frac{2Qa}{r^3}$ in a direction parallel from +Q to –Q

Electric Potential energy.

Electric potential energy is the energy that a charge has owing to its position in an electric field.



Consider a large positive charge A and a smaller positive charge B.
To move charge B from position 1 to position 2 requires work to be done against the electrostatic force pushing against B. The energy used in this work increases the electric potential energy that B now has.
This increase in energy can be recovered by charge B moving back to position 1.
Since an electric field extends to infinity, it is reasonable that charge B will have zero electric potential energy when it is at an infinite distance from charge A.

The force acting on charge B at position 1

$$F = E \bullet Q_t = \frac{Q Q_t}{4\pi \varepsilon_0 r^2}$$

If δr is very small compared to r, then E can be considered as constant as B moves from position 1 to 2.
{ $(r-\delta r)^2 = (r^2 - 2r\delta r + \delta r^2)$ If δr is very small, then compared to r^2, the other components can be ignored.}

Therefore the change in Electric Potential energy, δW, in going from position 1 to 2

$$\delta W = -\frac{Q Q_t}{4\pi \varepsilon_0 r^2}\delta r$$

The electric potential energy, W, that charge B has as a result of being at position 1

$$W = \int_\infty^r \delta W = -\int_\infty^r \frac{Q Q_t}{4\pi \varepsilon_0 r^2} dr = -\frac{Q Q_t}{4\pi \varepsilon_0}\int_\infty^r \frac{dr}{r^2} = \frac{Q Q_t}{4\pi \varepsilon_0}\frac{1}{r}$$

=> The electric potential energy that charge B has as a result of being at position 1

$$W = \frac{Q Q_t}{4\pi \varepsilon_0}\frac{1}{r}$$

Electric Potential

When work is done on a charge Q to bring it from infinity to a particular point in an electric field, it gains electric potential energy.
Electric Potential, V, is defined as the electric potential energy gained per unit charge

$$\Rightarrow V = \frac{W}{Q}$$

and has units of Volts.
So for charge B, its electric potential at position 1

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$$

(This particular result is only valid for a radial field.)
Therefore Electric Potential difference between two points in an electric field is the difference in potential between the two points.
An equipotential surface is a virtual surface in an electric field where the Electric potential is constant. Therefore a charge can be moved anywhere on an equipotential surface without any work being done.

Relationship between Electric Potential and Electric field strength.

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$$

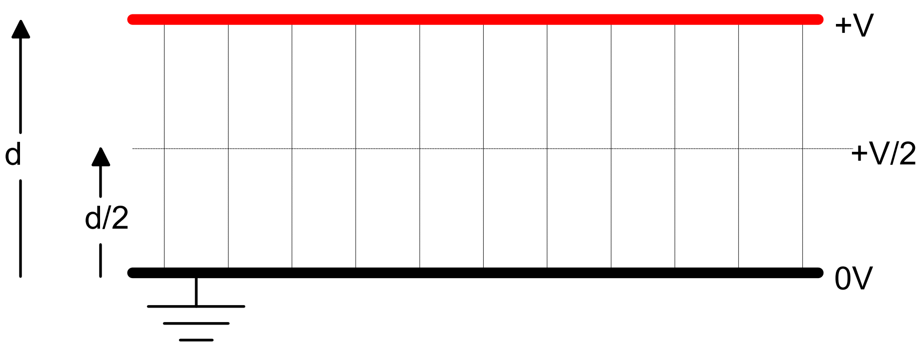
Potential gradient, how V changes with r

$$\frac{dV}{dr} = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} = -E$$

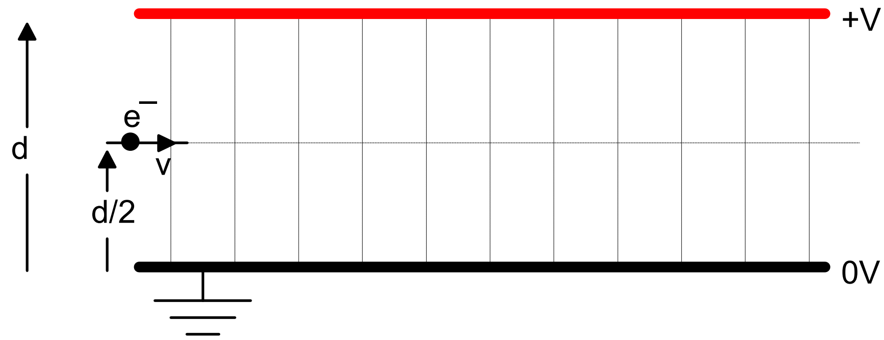
$$\Rightarrow E = -\frac{dV}{dr}$$

By definition, the units of E are NC^{-1}. This result also shows that Vm^{-1} are also valid units for E.

If E is uniform, then the electric potential will change linearly along the direction of the electric field strength.



Motion of a charged particle in a uniform Electric field.



Consider an electron, travelling with velocity v, passing into the space between two conducting plates with a potential difference of V.
There is no force acting horizontally and so the horizontal velocity is unchanged.
The force acting on the electron vertically: F = EQ, and is upwards.
=>The path of the electron is therefore an upwards parabola.

$E = V/d$ => $F = Ve/d$ =>vertical acceleration, $a = Ve/md$

Time taken for the electron to travel d/2 vertically $s = ut + \frac{1}{2}at^2$

$$\frac{d}{2} = \frac{1}{2}\frac{Ve}{md}t^2$$

$$\Rightarrow t^2 = \frac{md^2}{Ve} \Rightarrow t = d\sqrt{\frac{m}{Ve}}$$

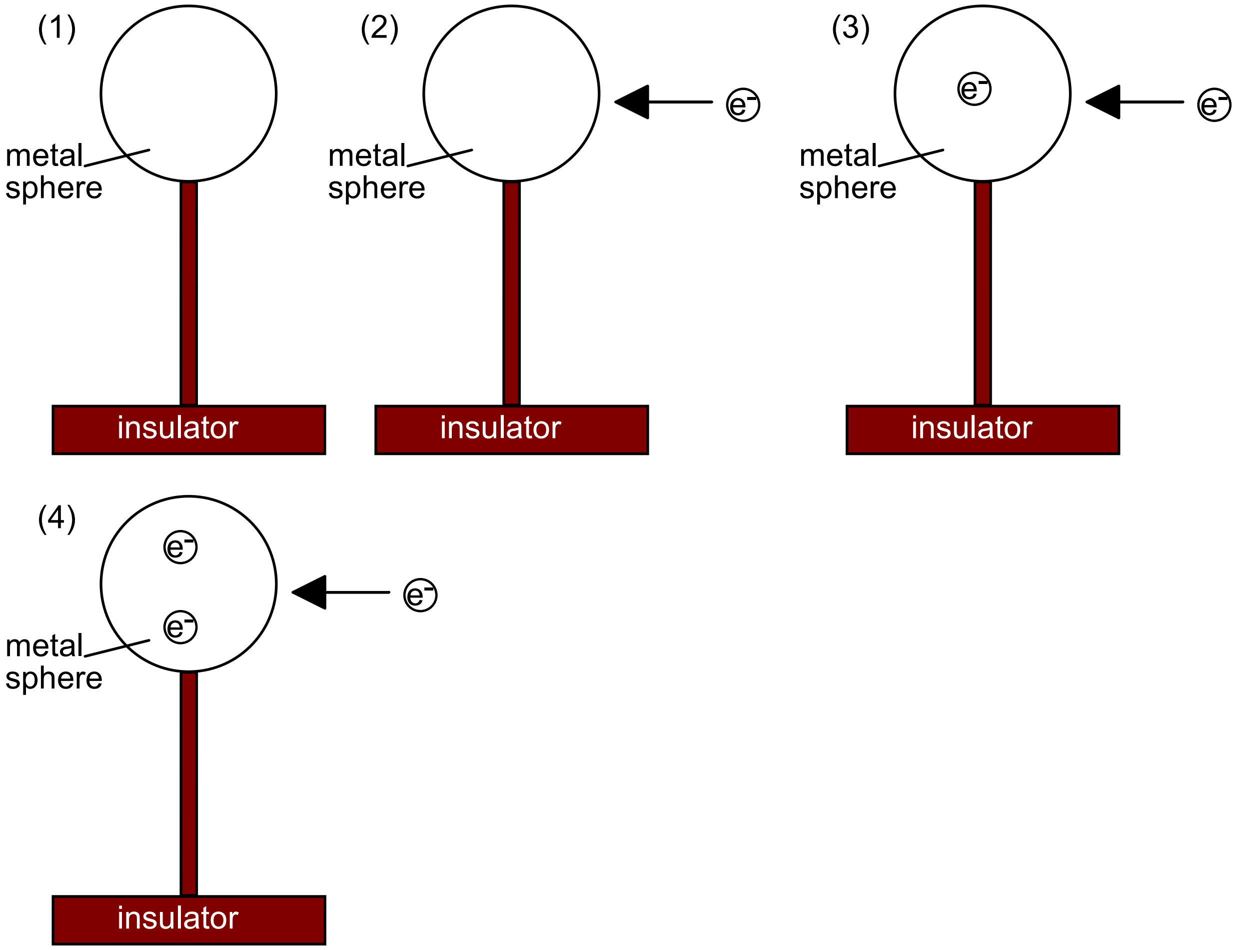
During this time, the electron travels a horizontal distance = v.t, so if v.t is shorter than the length of the top plate, it will collide with the top plate.

If $v = 10^7 ms^{-1}$, $V = 1000V$, $d = 5\times10^{-2}m$ and the plates are 100mm long, show that the electron collides $\approx$3.8cm along the positive plate.

How would this be different if an alpha particle replaced the electron?

Electric Capacitance

Consider the insulated uncharged metal sphere in (1).
In (2) an electron approaches the metal sphere. Due to charge separation in the metal sphere, there will be a small force of attraction acting on the electron. When the electron reaches the metal sphere it settles giving the sphere a negative charge.
In (3), a second electron approaches the negatively charged metal sphere. A repulsive force pushes against the electron, so work has to be done to move the electron to the metal sphere. When it reaches the metal sphere it settles, increasing the sphere's negative charge. The work done on the electron is stored on the metal sphere as electric potential energy, so increasing the electric potential of the sphere.



In (4), a third electron approaches the negatively charged metal sphere. The repulsive force is now stronger owing to the second electron on the sphere and so pushes against the approaching electron, increasing the work that has to be done to move the electron to the metal sphere. When it reaches the metal sphere it settles, increasing the sphere's negative charge and increasing the electric potential of the sphere.
If this process continues, the charge and electric potential of the sphere increase.

The ratio of the charge, Q, to electric potential, V is called **Capacitance**, C

$$C = \frac{Q}{V}$$

The unit of capacitance if the **Farad, F** => $F = CV^{-1}$.

The electric potential, V, at a distance, r, from a point charge, Q, $V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$

Outside of a charged metal sphere, the electric field is the same as that of a point charge placed at the centre of the sphere.
But Q = CV

$$\Rightarrow V = \frac{CV}{4\pi\varepsilon_0}\frac{1}{r}$$

$$\Rightarrow C = 4\pi\varepsilon_0 r$$

=> The capacitance, C, of a conducting sphere of radius, r, = $4\pi\varepsilon_0 r$

The earth has a radius of 6.4×10^6m and so it has a capacitance of
$C = 4 \times 3.142 \times 8.85 \times 10^{-12} \times 6.4 \times 10^6 = 7.1 \times 10^{-4}$F

As can be seen, the Farad is a very large unit and practical capacitors are measured in
microfarads, μF, 10^{-6}F and
picofarads, pF, 10^{-12}F

Earthing

Consider a conducting sphere of radius r, with an electrical potential of V_1.
The capacitance of this sphere = = $4\pi\varepsilon_0 r$
The charge stored on the sphere is $Q_1 = C_{sphere}V_1$

The earth is predominantly made of iron and so is conducting.
If the conducting sphere is now connected to the earth, the electric charge will be distributed so that the sphere and the earth have the same electric potential.
The total capacitance is the capacitance of the sphere, C_{sphere} + the capacitance of the earth, C_{earth}.
Charge is always conserved, therefore the electric potential of the earth and sphere, V, where

$$V = \frac{Q_1}{C_{sphere} + C_{earth}} = \frac{C_{sphere}V_1}{C_{sphere} + C_{earth}}$$

$$\Rightarrow \frac{V}{V_1} = \frac{C_{sphere}}{C_{sphere} + C_{earth}} = \frac{r_{sphere}}{r_{sphere} + r_{earth}}$$

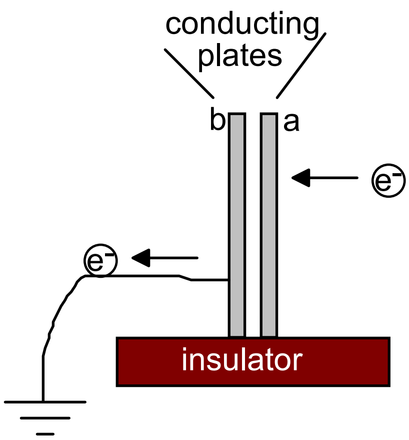
So if the conducting sphere had a radius of 0.1m and an electric potential of 1000V, after it has been connected to the earth its electrical potential will be ≈ 16μV.

In practice, the earth is considered to be the practical zero of electric potential and electrical connections to the earth are an essential part of the electricity distribution network and communications networks.

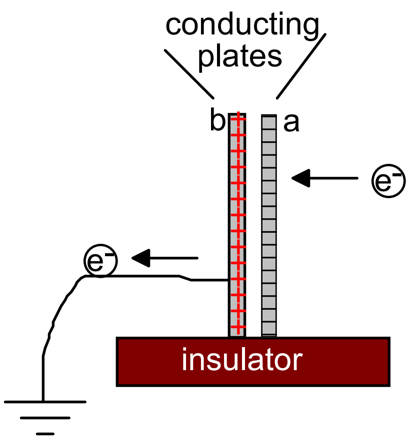
If such a sphere at a potential of 1000V is connected to earth a small spark and a large current will flow for a short time until the electric potential difference. The spark and large flow of current will result in electric potential energy being converted into heat energy and electromagnetic radiation.

Practical Capacitors

Insulated spheres make poor practical capacitors owing to their physically large size and small capacitance. Practical capacitors usually consist of two close conducting surfaces separated by an insulator; with one of the conducting surfaces being connected to earth.



When an electron approaches plate a, an electron is repelled from plate b towards earth.
When an electron is put onto plate a, it will have a negative charge; plate b will have lost an electron to earth and so be left with positive charge.



As each electron approaches plate a, more electrical work has to be done, and when the electron is finally added to plate a, the electrical work done is stored in the electric field between plates a and b.
The electric field strength, E, between the plates;

$$E = V/d,$$

where V is the difference in electrical potential between the plates and d is the distance between the plates.
Capacitance, C is defined as:

$$C=Q/V,$$

where Q is the charge on one of the plates.
So the capacitance of the two metal plates:

$$C = \frac{Q}{E \cdot d}$$

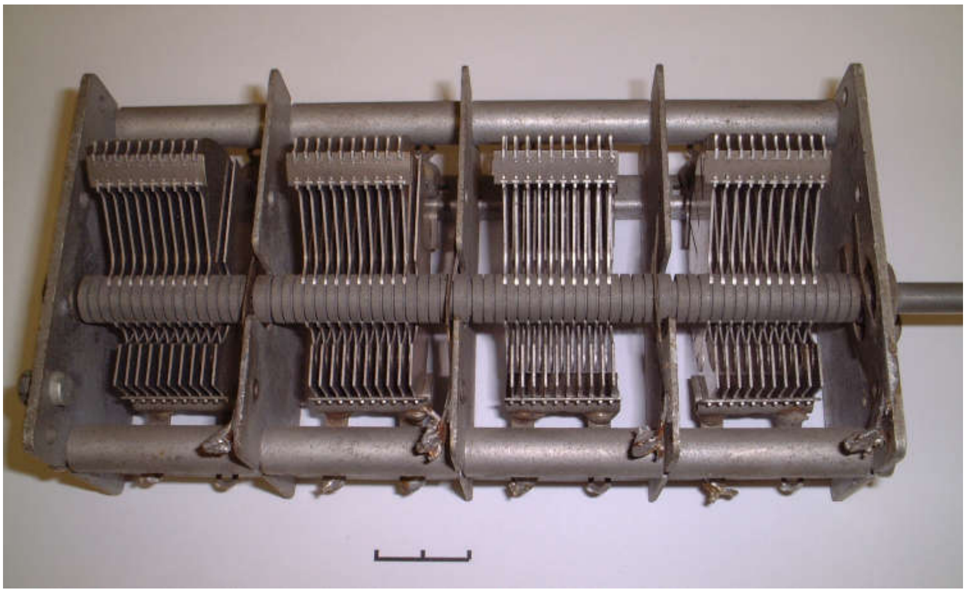
The electric field strength at a distance d from a point charge:

$$\Rightarrow E = \frac{Q}{4\pi \varepsilon_0 d^2} = \frac{Q}{A \varepsilon_0}$$

($4\pi d^2$ = the area, A, of a sphere, centred on the point charge, radius d).
This is valid for the conducting plates.

$$\Rightarrow C = \frac{A\varepsilon_0}{d}$$

Where A is the area of overlap of the metal plates and ε_0 is the permittivity of free space.
E.g. If two overlapping metal plates, 10cm by 10cm are separated by 1mm, calculate their capacitance.
$C = 0.1\times0.1\times8.85 \times 10^{-12}/10^{-3} = 8.85 \times 10^{-11}$ F = 88.5pF



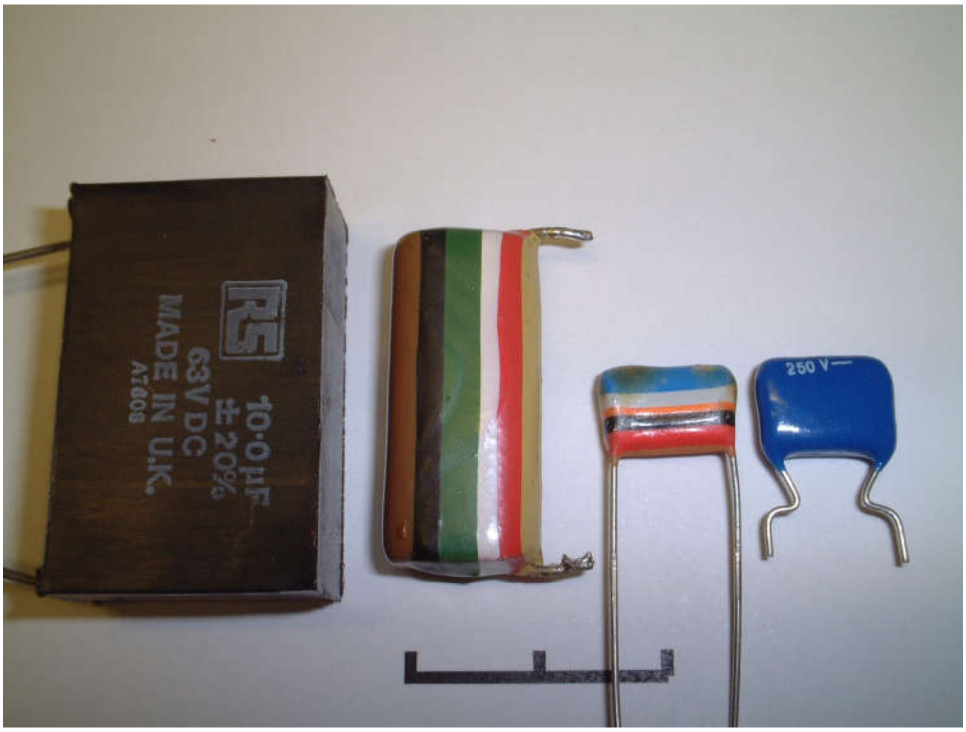
This is an example of a variable capacitor with a maximum capacitance of ≈2000pF

To increase the capacitance, A can be increased and d can be reduced.
Air, at STP breaks down at ≈ 3000Vmm^{-1}, which limits the maximum potential difference that can be used.
This can be substantially increased by replacing the air between the plates with an insulating material, e.g, Polystyrene, ≈ 60,000Vmm^{-1} and mica, ≈ 400,000Vmm^{-1}.
Insulators, used in this way are known as dielectrics, and as well as increasing the breakdown voltage, they also increase the capacitance.
The factor by which the capacitance is increased is called the Relative permittivity and given the symbol ε_r, and for polystyrene is ≈ 2 and mica, ≈ 5
The formula for the capacitance of a parallel plate capacitor with a dielectric becomes

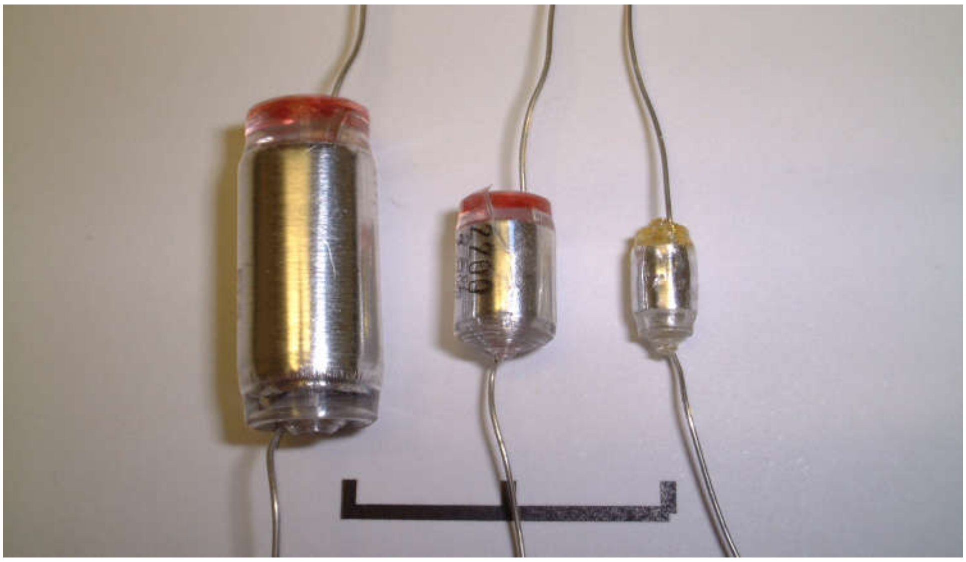
$$C = \frac{A\varepsilon_0 \varepsilon_r}{d}$$

With a dielectric, the size is significantly reduced - this having a maximum value of ≈600pF

Two parallel conducting plates have a very limited capacitance, even with a dielectric separator. These are fixed value capacitors, with many hundreds of metal plates, separated by a polyester dielectric.
The black one has a value of 10μF (10^7pF)



Capacitors can also be rolled up



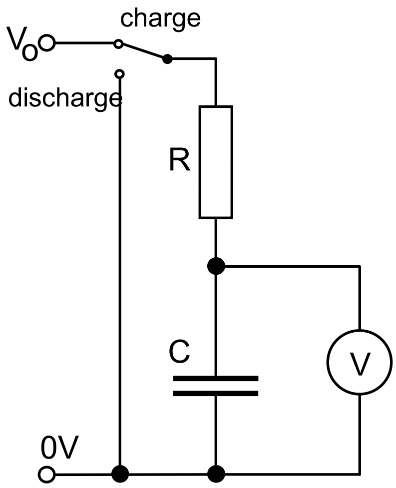
Very large value capacitors can be made by using aluminium oxide as the dielectric, which is chemically formed directly on the metal plates of the capacitor. These are known as Electrolytic capacitors and require a very small current to pass through the capacitor to maintain the aluminium oxide layer. As a result, they are polarised and have to be connected the correct way round in a circuit.

This one has a value of 0.47F, - compare with the earth at 7.1×10^{-4}F

Super capacitors (Ultra capacitors, Electric Double Layer Capacitors, EDLC) often use a carbon coating on the conducting plates to massively increase the surface area. These can have values of 100F+ and are often used in electric cars to store energy from regenerative braking.
They have huge power densities (10kW/kg)

Note the very large capacitance, but low voltage working.

Capacitor charging and discharging
Charging



Adding the potential differences:

$$V_0 = IR + V$$

But Q=CV

$$\Rightarrow V_0 = IR + Q/C$$

But I = dQ/dt

$$V_0 = \frac{dQ}{dt}R + \frac{Q}{C}$$

$$CV_0 - Q = \frac{dQ}{dt}RC$$

$$\int_0^t \frac{dt}{RC} = \int_0^Q \frac{dQ}{(CV_0 - Q)}$$

$$\left[\frac{t}{RC}\right]_0^t = \left[-\ln(CV_0 - Q)\right]_0^Q$$

$$-\frac{t}{RC} = \ln(CV_0 - Q) - \ln(CV_0) = \ln\left(\frac{CV_0 - Q}{CV_0}\right)$$

$$e^{-t/RC} = \frac{CV_0 - Q}{CV_0}$$

$$\Rightarrow Q = Q_0\left(1 - e^{-t/RC}\right)$$

since $Q_0 = CV_0$
Similarly

$$\Rightarrow V = V_0\left(1 - e^{-t/RC}\right)$$

As can be seen from this formula, the product of R and C has units of time.
The quantity RC is called the Time Constant.
The voltage across the capacitor after one time constant:

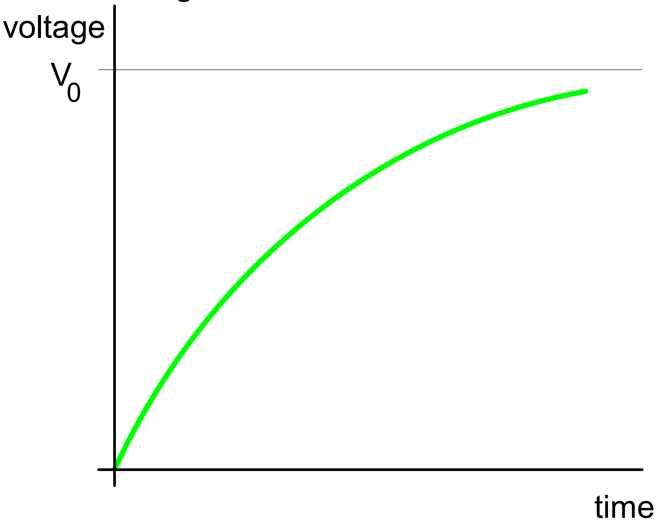
$$V = V_0(1 - e^{-1}) = V_0(1 - 0.368) = 0.632V_0$$

=> after one time constant, the capacitor has charged to ≈63% of the charging voltage.

To fully charge the capacitor takes a very long time.
It is normal to assume that the capacitor has fully charged after five time constants

$$t=5RC$$

by which time the capacitor has charged to 99.3% of V_0.



Differentiating this formula gives

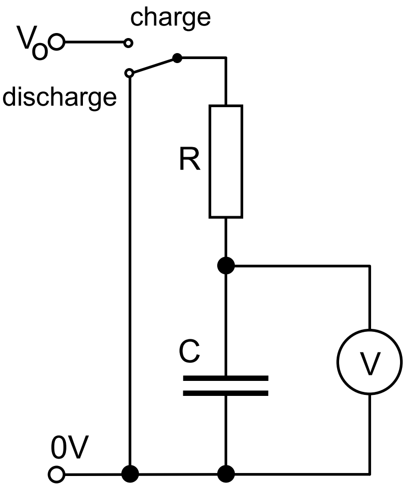
$$\frac{dV}{dt}=\frac{V_0 e^{-t/RC}}{RC}$$

Setting t = 0, will give the gradient of the graph at the origin

$$=\frac{V_0}{RC}$$

which enables the value of the capacitor to be calculated by measuring the gradient.

Discharging



When the switch is moved from the charge to discharge position, the capacitor begins to discharge through R. The supply voltage, V_0, has been replaced by 0V.

$$0 = IR + V$$

But Q=CV

$$\Rightarrow 0= IR + Q/C$$

But I = dQ/dt

$$\frac{dQ}{dt}R = -\frac{Q}{C}$$

$$-Q = \frac{dQ}{dt}RC$$

$$\int_0^t \frac{dt}{RC} = -\int_{Q_0}^{Q} \frac{dQ}{Q}$$

$$\left[\frac{t}{RC}\right]_0^t = \left[-\ln Q\right]_{Q_0}^{Q}$$

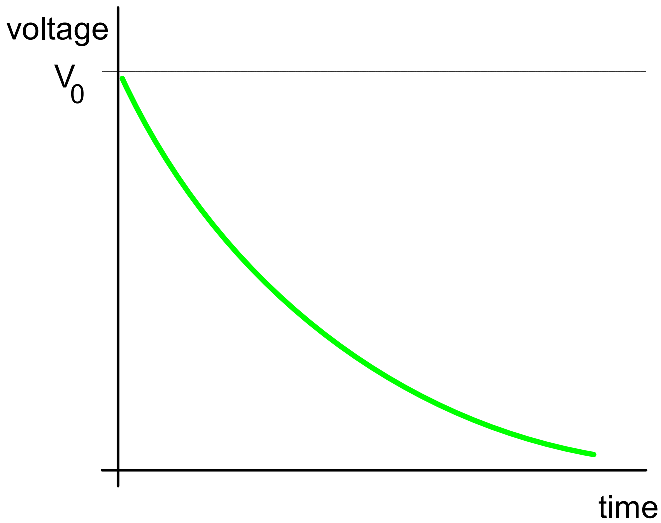
$$-\frac{t}{RC} = \ln(Q) - \ln(Q_0) = \ln\left(\frac{Q}{Q_0}\right)$$

$$e^{-t/RC} = \frac{Q}{Q_0}$$

$$\Rightarrow Q = Q_0 e^{-t/RC}$$

since $Q_0 = CV_0$ and $Q = CV$

$$\Rightarrow V = V_0 e^{-t/RC}$$



The voltage across the capacitor after one time constant:

$$V = V_0 e^{-1} = V_0 0.368$$

=> after one time constant, the capacitor has discharged to ≈37% of the initial voltage.
To fully discharge the capacitor takes a very long time.
It is normal to assume that the capacitor has fully discharged after five time constants

$$t=5RC$$

by which time the capacitor has discharged to ≈ 0.7% of V_0.