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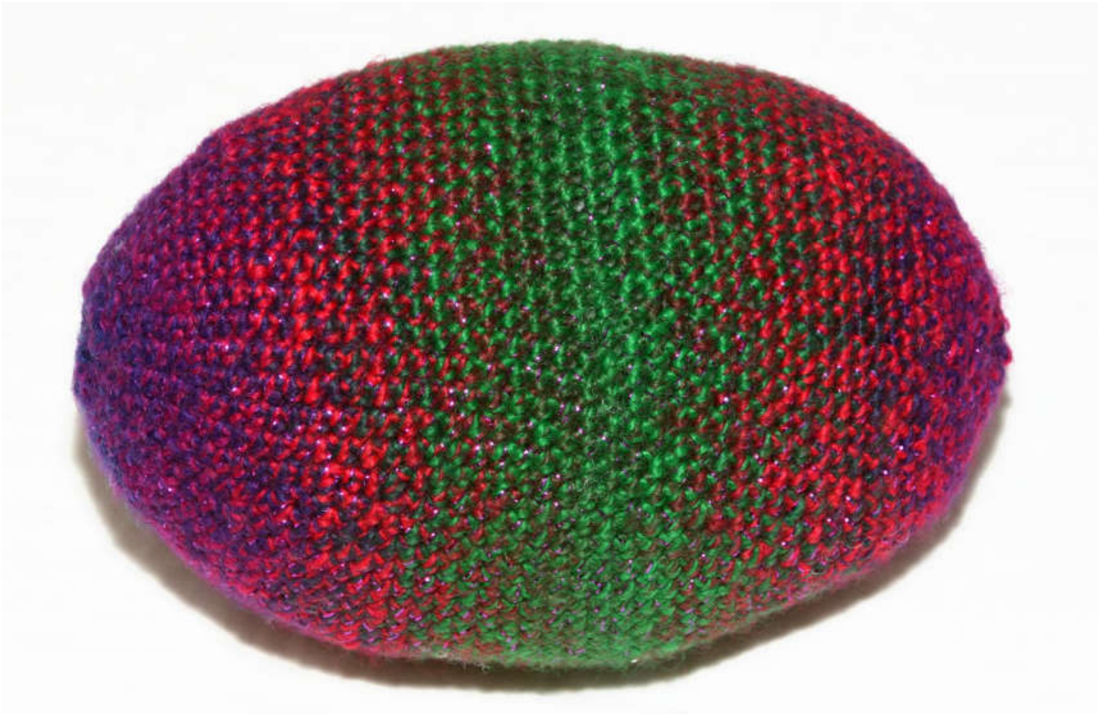
Crochet Ellipsoids

Introduction

How many stitches should there be in each round of crochet to produce an ellipsoid?
There are web sites on the Internet which give details for a specific size made with a specific size of wool, hook and tension, but there seems to be nothing of a general nature.



A 100mm x 150mm ellipsoid with DK yarn.

It is clear that the number of stitches will depend on the number of stitches per mm, the height of each round and the size of the ellipsoid.
With this information, a formula can be produced for the number of stitches per round and its derivation is given under the Theory heading.
The width of a stitch, w and the height of a stitch, h are found by making a 'test square' of fabric. This should be at least 15 stitches by 15 rows to be reasonably accurate.

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Making the calculations directly is tedious and so a web page calculator has been created to do this.

http://www.ikes.16mb.com/physproj/Ellipsoid_crochet.htm

The calculator also suggests a suitable pattern for creating the ellipsoid which aims to produce the best 'ellipsoid' possible by evenly distributing the increases and decreases throughout each round.

NOTE

If the long and short axis are given the same measurements, then the result will be a sphere.

Abbreviations.

The crochet patterns use the English notation.

inc = increase is made by making two stitches into the same stitch.
rnd = round
× = repeat so many times
e.g. (1st, inc)×6 means (crochet 1 stitch, then make 2 stitches in the next stitch) and repeat this 6 times in total.
ss = slip stitch
st = stitch. The type of stitch is not specified and will depend on what was used to form the tension sample.
loop method = This is used for the first round.
Make a loop of yarn, with the tail hanging downwards and the working yarn overlapping in front of the tail. Holding the loop in place, insert the hook through the centre of the loop and pull the working yarn through the loop.
Make chain(s). Work the requires number of stitches into the loop and then pull the loop tight to complete the first round.
FO Finish off, by cutting the yarn leaving a tail for sewing in and then pulling through the last stitch.

It will be found helpful to mark the beginning of each round with a piece of different coloured yarn. This makes it easier to correct any miscounting of stitches that may occur.

The ellipsoid will need to be stuffed before completing the crochet.
It is recommended that this is done when there are just three or four more rounds left to complete.
When the last round has been completed, the stuffing can be topped up if necessary and the crochet fastened off by cutting the yarn, leaving a long tail, and pulling it through the last stitch.
The tail can then be threaded through the stitches of the last round with an embroidery needle.
The tail can then be pulled tight to close up the sphere and the tail sewn into the ellipsoid to secure.

Theory.

2a = length of the smaller axis (along x axis)
2b = length of the larger axis (along y axis)
t = ellipse parametric equation variable
h = height of a stitch
w = width of a stitch
n = Round number

Determining w and h.

Crochet/knit a tension test of at least 15st x 15 rows, using the chosen wool, pattern and size of hook/needles.
(Double knit wool with a 4.0mm hook works well, producing a close fabric that will take being stuffed.)

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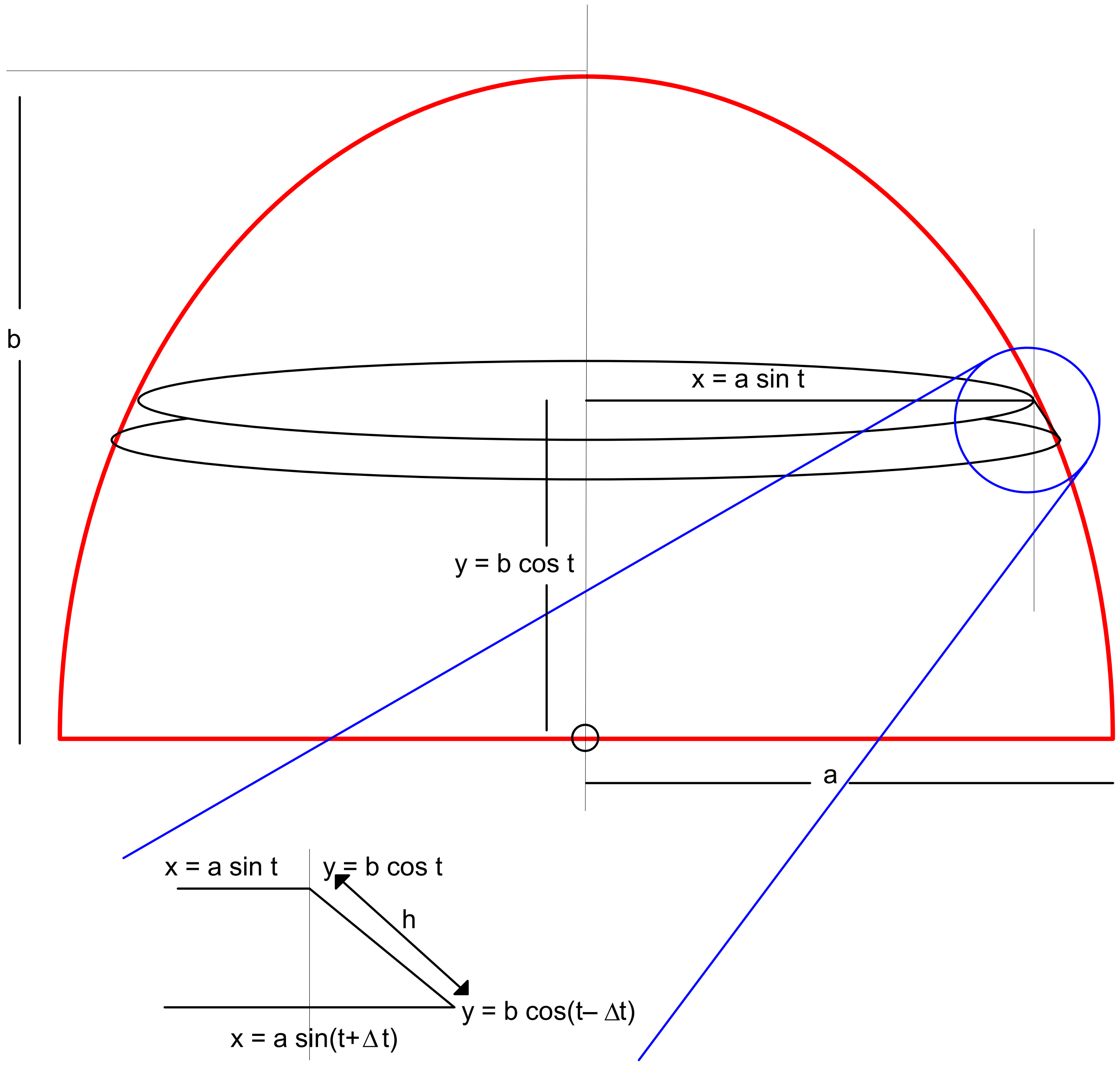
Ellipse equations

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

For this application, the parametric equations of an ellipse are
$x = a \sin t$ $\quad$ $y = b \cos t$
This is because the calculations begin at the top of the ellipse when x=0.

NOTE t is NOT the angle at the centre



where h is the 'height' of a stitch

From Pythagorus
$h^2 = a^2(\sin(t+\Delta t) - \sin(t))^2 + b^2(\cos(t) - \cos(t-\Delta t))^2$

Simplifying with trig identities

$h^2 = a^2(2\cos(t+\Delta t/2).\sin(\Delta t/2))^2 + b^2(-2\sin(t-\Delta t/2).\sin(\Delta t/2))^2$

$h^2 = 4\sin^2(\Delta t/2)(a^2\cos^2(t+\Delta t/2) + b^2\sin^2(t-\Delta t/2))$
$\Rightarrow h = 2\sin(\Delta t/2)\sqrt{(a^2\cos^2(t+\Delta t/2) + b^2\sin^2(t-\Delta t/2))}$

Ideally, this should be re-arranged to make Δt the subject, but an approximation can be made by using the values of t and Δt from Round n to find Δt for round n+1.

$$\Rightarrow \sin(\Delta t_{(n+1)}/2) = h/(2\sqrt{(a^2\cos^2(t_n+\Delta t_n/2) + b^2\sin^2(t_n-\Delta t_n/2))}$$

$$\Rightarrow \Delta t_{(n+1)}) = 2\sin^{-1}(h/(2\sqrt{(a^2\cos^2(t_n+\Delta t_n/2) + b^2\sin^2(t_n-\Delta t_n/2)))})$$

For Round n, the radius (x value) = $a \sin t_n$
The circumference around this Round = $2 \pi x = 2 \pi a \sin t_n$
=> the number of stitches in Round n = $2 \pi a \sin t_n / w$,
where w is the width of a stitch

The Pattern.
Round 1
For sensible values of a and b, the top of the ellipsoid is reasonably parallel with the x axis.
$\Rightarrow x_1 = a \sin t_1 = h, \quad \Delta t_0=0$
$\Rightarrow t_1 = \sin^{-1}(h/a)$
=> Δt_1 can now be calculated.
The number of stitches in Round 1 = $2 \pi h/w$

Round 2
$\Rightarrow x_2 = a \sin(t_1 + \Delta t_1)$
The number of stitches in Round 2 = $2 \pi x_2/w$
=> Δt_2 can now be calculated.

This continues while $x_n > h$

Trig identities

$\cos(A+B) = \cos A.\cos B - \sin A.\sin B$
$\cos(A-B) = \cos A.\cos B + \sin A.\sin B$

$\Rightarrow \cos(A+B) - \cos(A-B) = -2\sin A.\sin B$
$\Rightarrow \cos(A+B) + \cos(A-B) = 2\cos A.\cos B$

$\sin(A+B) = \sin A.\cos B + \cos A.\sin B$
$\sin(A-B) = \sin A.\cos B - \cos A.\sin B$

$\Rightarrow \sin(A+B) - \sin(A-B) = 2\cos A.\sin B$
$\Rightarrow \sin(A+B) + \sin(A-B) = 2\sin A.\cos B$