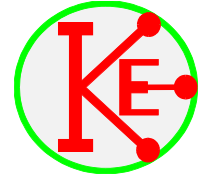# Basic Electricity 2

Electricity in solids

Electrically, solids can be classified as good conductors, poor conductors and non-conductors of electricity.

Good conductors, often just called **Conductors**, allow electrons to move freely.

Poor conductors, often called **Semiconductors**, do not allow electrons to move freely.

Non-conductors, often called **Insulators**, allow no movement of electrons.

Matter is made from atoms, which consist of a central nucleus and surrounding electrons.
The outer electrons are shielded from the nucleus by those closer to the nucleus and it is these outer electrons that are responsible for the electrical properties of solids
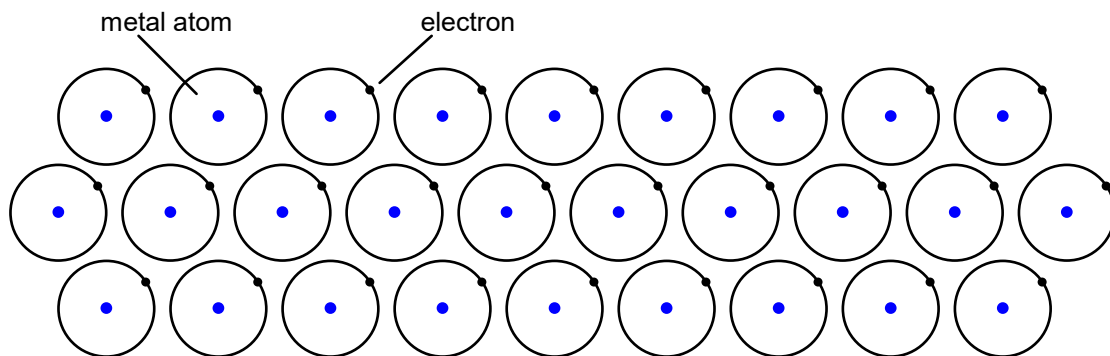
Conductors

All metals are good conductors of electricity. The only non-metal that is a good conductor at room temperature is Graphite (Carbon).
Neutral atoms have the same number of electrons and protons, so that there is no net electric charge.
A metal atom has one or more outer electrons that are weakly held in place by the nucleus.
In a solid, the metal atoms are fixed in place, but the outer electrons are so weakly attracted to the atoms that at normal temperatures they have sufficient thermal energy to move 'freely' around the solid. These electrons are often referred to as **Conduction electrons**.

In a conductor, the number of conduction electrons is large.
Copper metal atoms have one outer electron that can move around.
One mole of copper atoms has a mass of 6.35×10^{-2}kg.
So 1 mole (6.023×10^{23}) of copper atoms have a mass of 6.35×10^{-2}kg.
The density of copper is 8.9×10^{3}kgm^{-3}.

So in one m^3 there are $\dfrac{6.023 \times 10^{23} \times 8.9 \times 10^{3}}{6.35 \times 10^{-2}} = 8.44 \times 10^{28}$ conduction electrons.

The conduction electrons move very quickly.
Their speed can be estimated by equating their kinetic energy to their thermal energy.

$$=> \ \tfrac{1}{2}mv^2 = kT$$

where k is the Boltzmann constant, T is the absolute temperature, m is the mass of an electron and v is the velocity of an electron.
Substituting in values gives an approximate velocity of 10^5ms^{-1}.

If a potential difference is put across the metal, say from a battery, then the conduction electrons will drift along the wire from the negative side and towards the positive side of the battery. Electrons will enter the metal from the negative side of the battery and leave the metal to return to the battery at the positive side. So an electric current passes and the metal has not net charge.

The speed that the conduction electrons drift along a conductor is low.
Consider a length, d of wire of cross-sectional area, A, and made from a metal with n freely moving conduction electrons per cubic metre.
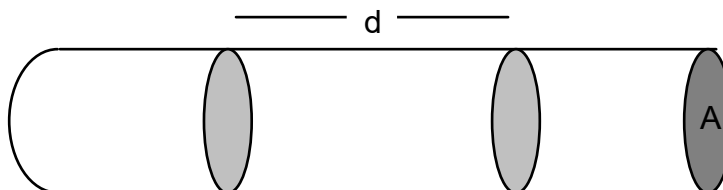
The volume of wire in the length d is = d×A
So the number of conduction electrons in this volume is = d×A×n
If e is the charge on the electron, then the moveable charge in this volume is = d×A×n×e
If it takes time t for this charge to move along and out of this volume, then since current = Q/t,

then $I = \dfrac{d \times A \times n \times e}{t}$

But speed = distance/time, so d/t can be replaced with v, the drift speed of the electrons
$$\Rightarrow I = v\,A\,n\,e$$
For copper, if I = 1 amp and A = 1mm^2 then
$$1 = v \times 10^{-6} \times 8.44 \times 10^{28} \times 1.6 \times 10^{-19}$$
$$\Rightarrow v = 7.4 \times 10^{-5} \text{ms}^{-1}$$

which is around 0.1mm per second!

However, when a light switch is operated, the lamp appears to respond immediately; there is no significant delay. Before the switch is operated, the connecting wire has a significant number of conduction electrons. When the switch is operated, extra electrons start to flow into the wire. These extra electrons exert a repulsive force on the conduction electrons, so pushing some of the conduction electrons along the wire and lamp, making the lamp light.
This is the same principle as when a hose pipe full of water is connected to a tap. When the tap is turned on, water appears to immediately come from the far end of the pipe. But the water coming out of the pipe is not the same as the water entering the pipe.

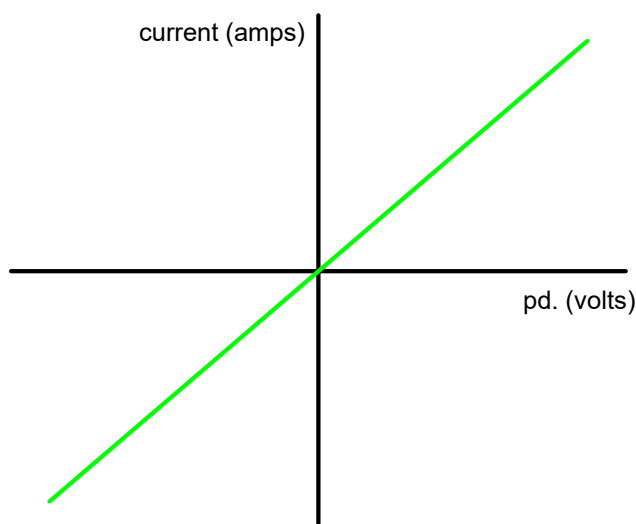electron
enters

electron
pushed out
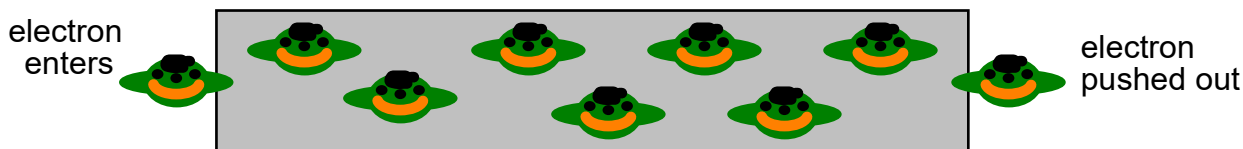
Resistance

As conduction electrons move in a metal when a current flows, they interact with both themselves and with the fixed metal ions. This impedes their progress and causes resistance. (Consider trying to move around a crowded shop!!)
This leads to a loss in energy which often results in an increase in temperature of the conductor.

Georg Ohm found that the current through a wire was proportional to the pd. across the wire so long as the temperature of the conductor did not change.
So a graph of current v pd. is a straight line, i.e. the wire has a constant resistance.
(The pd. is the independent variable and so on the 'x axis', - for without a pd., there is no current.)

current (amps)

pd. (volts)

If the temperature increases, then two things happen.
The outer electrons gain more thermal energy and the metal atoms vibrate with a larger amplitude.
Since most outer electrons are already free to wander, they just move around quicker.
Since the metal atoms are now occupying a larger volume, they interact more with the conduction electrons, so opposing the motion of these electrons and increasing the resistance of the metal.

This can be seen by measuring the current passing through a filament lamp as the pd. is varied.
The temperature of the metal filament will change from room temperature (e.g. 20°C) to over 1000°C when the lamp is switched on
The graph below reflect this increase in temperature.

current (amps)

pd. (volts)
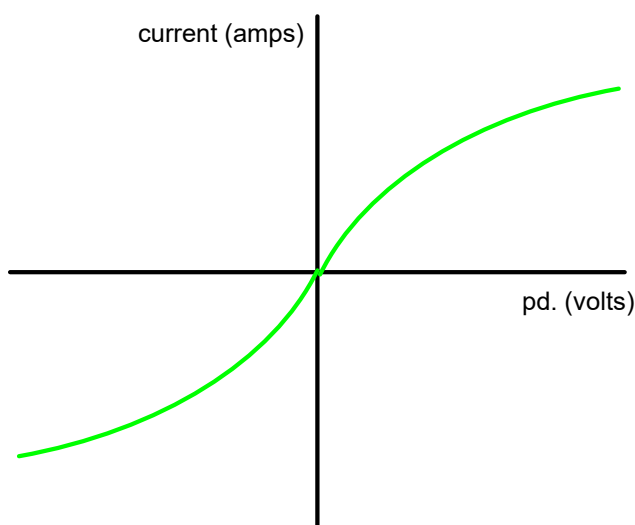
The more opportunity that the electrons moving in a current have to interact with both themselves and the fixed metal ions, the greater the loss in energy and the larger the resistance.

A long wire will give more opportunities for interaction than a than a short wire and so a long wire will have a larger resistance than a shorter wire.

Consider the long wire to be made from shorter lengths of wire all connected in series.

But we already know that for resistors in series, the total resistance is equal to the sum of the individual resistances.

So the resistance should be proportional to the number of shorter lengths of wire.

Experiments confirm that the resistance of a uniform wire is proportional to the length, ℓ. $R \propto \ell$

Similarly, a thick wire should give less opportunity for the electrons moving in a current to interact, since they can be spread out across the cross sectional area of the wire.

Consider the thick wire to be made from thinner pieces of wire all connected in parallel.

But we already know that for resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

If there are n thinner pieces of wire then

$$\frac{1}{R_T} = \frac{n}{R_1} \Rightarrow R_T = \frac{R_1}{n}$$

Since the cross sectional area is proportional to the number of wires, the total resistance should be proportional to one over the cross sectional area of the wire.

Experiments confirm that the resistance of a uniform wire is inversely proportional to the cross sectional area, A $R \propto 1/A$

These two properties can be combined together to give $R \propto \ell/A$

Experiments confirm this, but the constant of proportionality is dependent on the material making up the wire and temperature. The constant of proportionality has the symbol ρ (rho) and is a property of the material.

$$R = \frac{\rho \times \ell}{A}$$

Re-arranging

$$\rho = \frac{A \times R}{\ell}$$

Resistivity has the units of **ohm.metres (Ωm)**.

Resistivity will vary with temperature and for copper is $1.7 \times 10^{-8} \Omega m$ at room temperature.

Materials can be roughly classified according to their resistivity.

conductors	10^{-2} to $10^{-8} \Omega m$
semiconductors	10^{-6} to $10^{6} \Omega m$
insulators	10^{11} to $10^{19} \Omega m$

A term related to resistance is **Conductance** with the symbol **G**.

Conductance = 1 / resistance and has units of **Siemens (S)** or sometimes **mohs.**

$$G = \frac{1}{R} = \frac{I}{V}$$

Similarly, **Conductivity** = 1 / resistivity and has a symbol of **σ**.

The units of conductivity are Siemens/metre Sm^{-1}.

EMF and Internal resistance

Ohm's law tells us that I = V/R.

If an ammeter with a very low resistance (0.1Ω) is connected across a fresh AA cell with a potential difference of 1.5V, then we might expect a very large current of 15A to pass through the ammeter. In practice, the ammeter is unlikely to indicate much more than 2 - 3 amps.

It is as if there is an 'internal resistance' within the actual cell itself, which is limiting the maximum current.

The internal construction of a cheap, non-rechargeable cell is shown below.
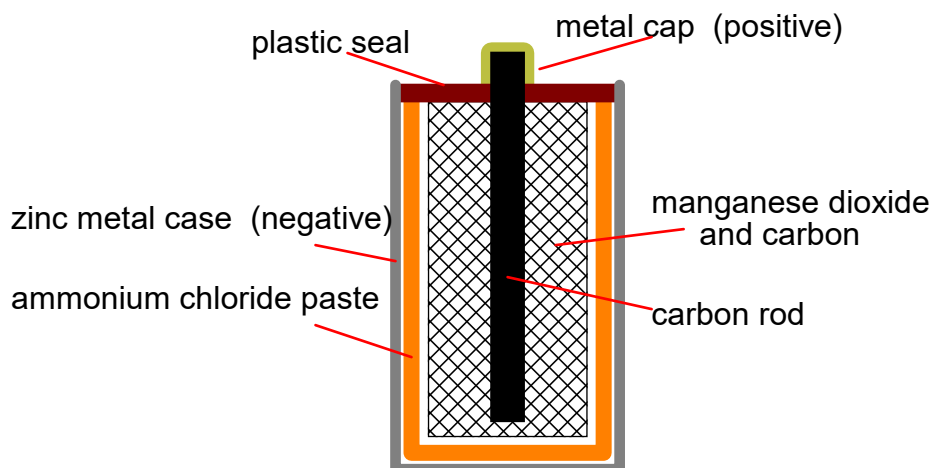

Looking at the cells construction, it is hardly surprising that there is an internal resistance!

In this cell, the zinc metal case reacts with the ammonium chloride paste liberating electrons. These electrons can flow through an external circuit to the metal cap and carbon rod where they help the manganese dioxide react with the ammonium chloride.

The energy given to the electrons in these reactions give a potential difference of around 1.55V.

The cell (or any other source of electricity) can be considered to be made of a perfect cell in series with a resistor (the internal resistance). Its circuit diagram becomes:-
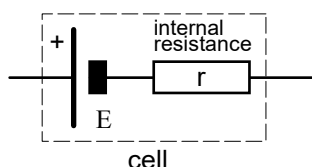
The pd. across the perfect cell is called the **Electromotive Force** (EMF) and is given the symbol, **E**.

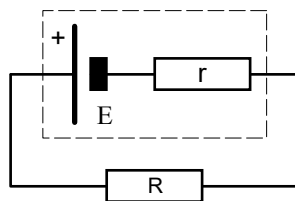The internal resistance is given the symbol **r**
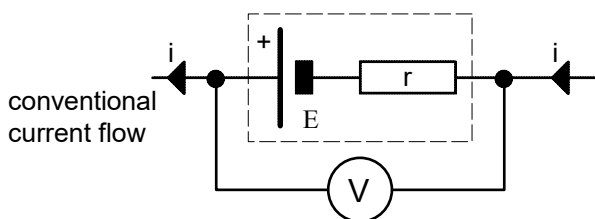
If a conventional current, i, is passing from the cell, then the pd. across the internal resistance will be i × r.

Therefore, the terminal pd. of the cell, V, will be

$$V = E - i \times r.$$

In the circuit below, a resistor, R, is connected across a cell.

The total resistance of the circuit is = R + r.

=> The current flowing in the circuit is $= \dfrac{E}{R + r}$

=> The pd. across R is $= \dfrac{E \times R}{R + r}$

=> This is the same as the pd. across the cell, V.

=> $V = \dfrac{E \times R}{R + r}$

Measuring EMF and internal resistance.

Modern digital voltmeters have a resistance of at least $1M\Omega$ and often much larger.
When such a meter is connected on its own across a cell, the current passing is so small that it can be ignored. The voltage across the internal resistance can therefore be ignored and the meter will indicate the EMF of the cell.

To measure the internal resistance of a cell, connect a resistor (e.g. 10Ω) across the cell and measure the pd. across the terminals of the cell.
If this terminal pd. is almost the same as the EMF, then use a smaller value resistor.
NOTE: Do not leave the resistor connected for too long as it could become quite hot!

The internal resistance can now be calculated by rearranging $\quad V = \dfrac{E \times R}{R + r} \quad => r = \dfrac{E \times R}{V} - R$

The internal resistance of a cell is not constant and changes with use and current supplied.
When a cell supplies a large current, gas bubbles form around the electrodes which increases the internal resistance of the cell. Leaving a cell to rest or warming it allows the gas bubbles to be absorbed, so reducing the internal resistance.
When a cell supplies a large current, the power dissipated in the internal resistance causes the cell to become hot, which may cause them to leak or even explode!

The power, P, supplied by a cell is $\quad P = \dfrac{V^2}{R} = \dfrac{(E \times R)^2}{R \times (R+r)^2} \Rightarrow P = \dfrac{E^2 \times R}{(R+r)^2}$

The maximum power that can be supplied by a cell can be found by differentiating P with respect to R, and then setting the differential to 0.

$$P = E^2 R \times (R+r)^{-2} \Rightarrow \dfrac{dP}{dR} = E^2 (R+r)^{-2} - 2E^2 R (R+r)^{-3}$$

If dP/dR is set to 0 (to find the maximum value) then $\quad 0 = 1 - 2R(R+r)^{-1} \Rightarrow R = r$

So the maximum power that a cell can deliver is when the load resistance is equal to the internal resistance. This is true for all sources of electrical power.

Insulators and semiconductors

If a material has no free electrons, then no current is able to flow.

Any electrons that are put onto the material will remain where they are put, which leads to the concept of Static Electricity. (Stationary electrons.)

Pure Carbon, in the form of diamond, is an excellent insulator.

The carbon atoms have 4 electrons in the outer part of the atom, and at room temperature, each of these electrons pairs with an outer electron of another carbon atom.

The diagram below represents this in 2-dimensions. In reality, diamond is a 3-dimensional crystal.
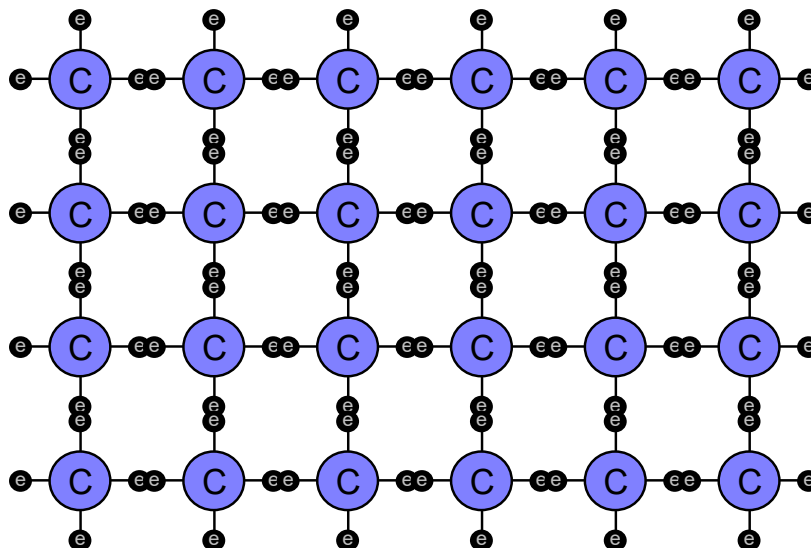

The pairs of electrons form strong covalent bonds, and it requires a temperature of around 5000°C before, on average, these electrons have sufficient energy to become 'free'.

The resistivity of diamond at room temperature is around $10^{16}\Omega m$.

Pure Silicon has a crystal structure similar to carbon but the electrons do not form such strong covalent bonds, and electrons, on average, can become 'free' at a temperature of around 350°C.

This means that at room temperature, there are a few electrons with sufficient energy to break free of the covalent bonds and move around.
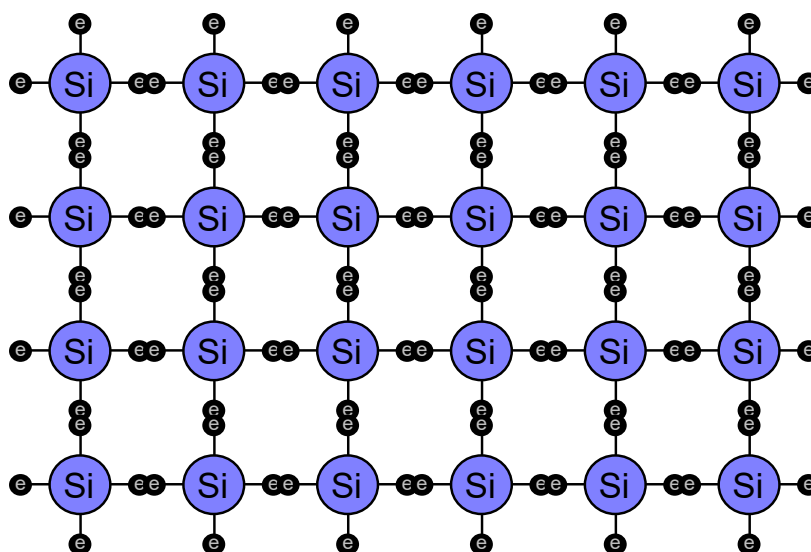
Silicon is therefore not such a good insulator as diamond.

The resistivity of pure silicon at room temperature is around $10^{4}\Omega m$.

If there are impurities in the silicon, then the situation changes significantly.
The concept of the Field Effect Transistor (FET, on which modern digital computers are based) was patented in Britain in 1935 by Oskar Heil, but could not be built because silicon could not be made with sufficient purity.

Consider adding a phosphorus atom to a silicon crystal.
Phosphorus has 5 electrons in the outer part of the atom.
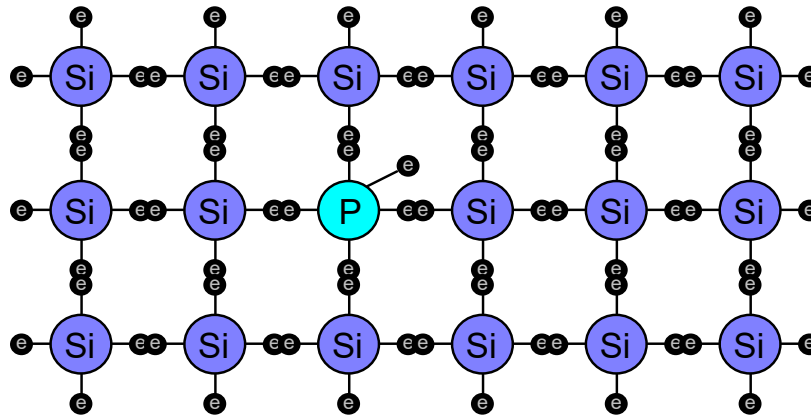
Four of the phosphorus electrons form covalent bonds with the other silicon atoms. The fifth electron is only weakly held in place by the phosphorus atom and so at room temperature is effectively free to move.
Silicon treated with impurities in this way is said to be 'doped'. If the doping produces electrons that can move, the material is called an **n-type** semiconductor.

Consider adding an aluminium atom to a silicon crystal.
Aluminium has 3 electrons in the outer part of the atom.
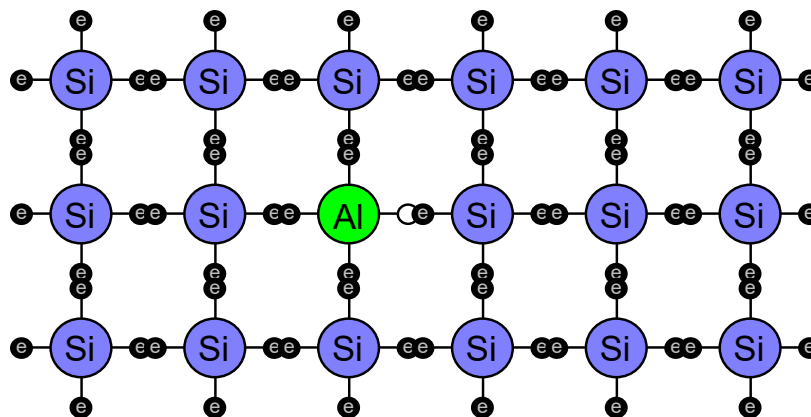
The three electrons from the aluminium atom form covalent bonds with three electrons from the other silicon atoms. This leaves a gap or **'hole'** into which an electron from elsewhere in the silicon crystal can move, so leaving a 'hole' elsewhere. In this way, the 'hole' is effectively free to move.
Silicon doped in this way is called an **p-type** semiconductor.

The amount of impurity added is very small and ranges from 1 part per 10^{12} to 1 part in 10^6
The resistivity of doped silicon ranges from around 10Ωm to $10^{-6}\Omega$m.
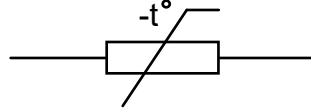
Thermistors

Doped silicon has several important uses including devices called **Thermistors**.
A thermistor is a device whose resistance varies with temperature.
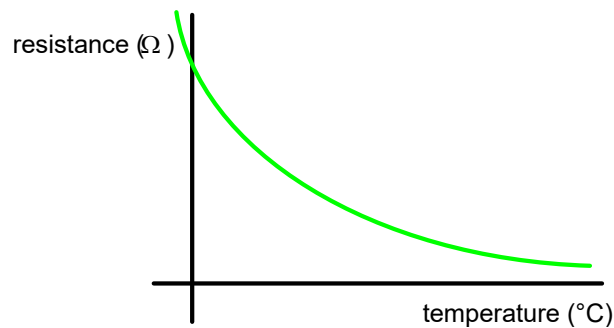There are two main types:-

ntc thermistors whose temperature decreases as temperature increases,
(negative temperature coefficient) and have an electrical symbol of

These are often made from lightly doped semiconductors, so that as the temperature increases, more electrons become 'free' and so the resistance decreases.
The range of resistance can be altered at manufacture by changing the amount and type of doping agent.
A graph of resistance v temperature is non-linear and has the form

ptc thermistors whose temperature increases as temperature increases,
(positive temperature coefficient) and have an electrical symbol of
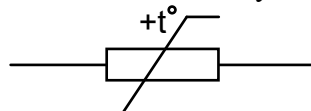
These are often made from highly doped semiconductors. As the temperature increases, the effect of the additional 'free' electrons is small compared to number of interactions of the electrons with the more vigorously vibrating ions, so making the resistance increase.
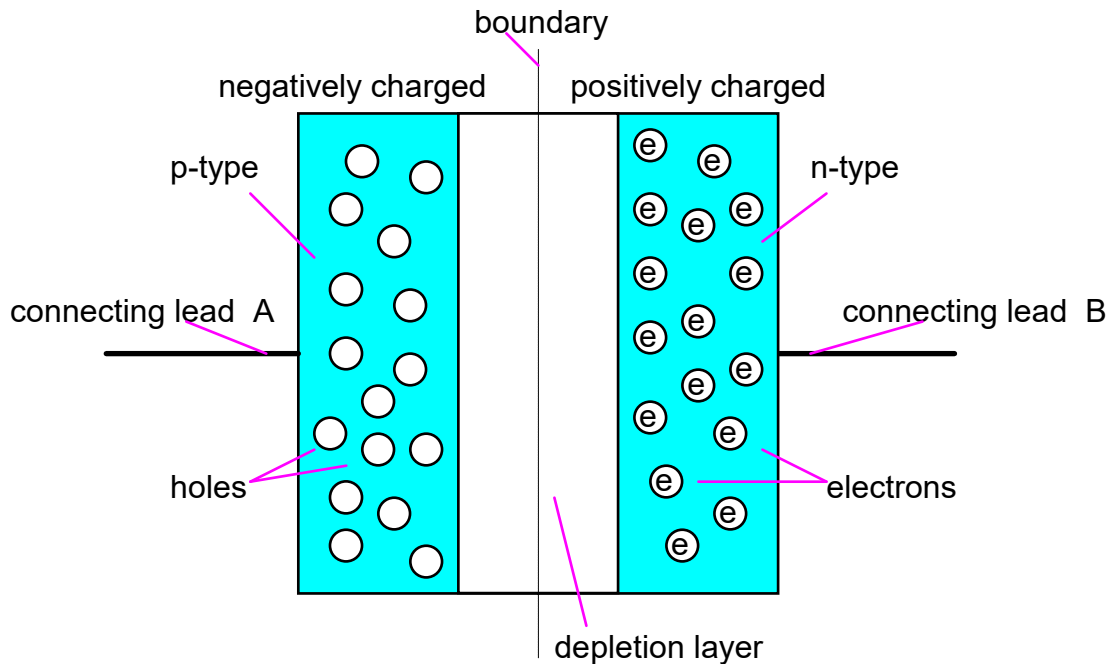The range of resistance can be altered at manufacture by changing the amount and type of doping agent.
A graph of resistance v temperature is usually non-linear and has the form
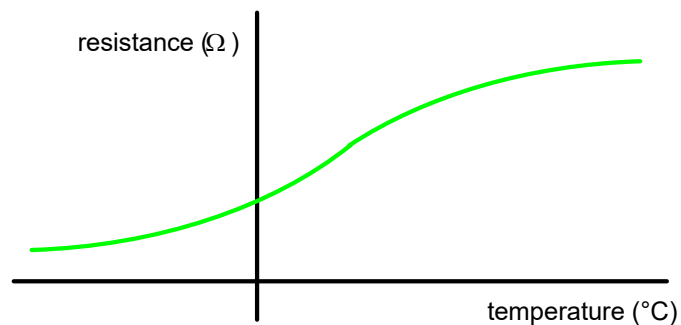
The p-n junction
Semiconductors become particularly interesting when a p-type and n-type are formed together to produce a junction.

Initially, the p-type and n-type semiconductors have no net charge.
When the junction is formed, electrons from the n-type move into the p-type to fill the holes.
This results in the p-type material becoming negatively charged (because it has gained electrons) and the n-type material becoming positively charged (because it has lost electrons).
The negative charge of the p-type repels any further electrons from moving across the boundary.
A region is left around the boundary where there are no holes or electrons, i.e. no free charge carriers.
This region is known as the **Depletion Layer** and is an insulator, i.e. no current can flow.

If a pd. is put across the p-n junction, with connecting lead A negative with respect to connecting lead B, then a few electrons will be attracted into the p-type material from the power source and fill holes. This makes the depletion layer get wider. Similarly, with the n-type material and a few electrons will be attracted into the power source, again making the depletion layer wider. Since there are still no free charge carriers in the depletion layer, no current can pass.
In this state, the p-n junction is said to be **reversed biased**.
As the pd. is increased, the depletion layer becomes wider and wider, until eventually the electrons from the power supply have sufficient energy to jump across the depletion layer, causing a large current to pass and destroying the p-n junction as the current causes it to become very hot.
The maximum pd that can be put across a p-n junction depends on how it is made and can vary from a few volts up to many kilovolts.

If a pd. is put across the p-n junction, with connecting lead A positive with respect to connecting lead B, then holes in the p-type material will be repelled into the depletion layer, reducing its width.
Similarly, electrons will be repelled from the n-type material into the depletion layer, reducing its width.
If the pd. is increased, the depletion layer will eventually disappear and then a current can freely pass through the junction. For silicon, the pd. needed is around 0.7V.
In this state, the p-n junction is said to be **forward biased**.

A p-n junction used in this way forms a device called a **Diode**.
The symbol for a diode is

conventional current

anode   cathode
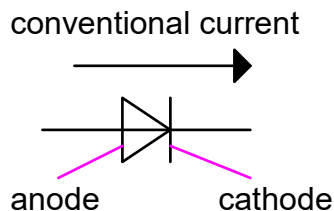
The diagram below shows the current v pd. graph for a typical silicon diode.
NOTE the different scale on the positive and negative pd. axis

current (amps)

reversed bias   forward bias

break down

-30   -20   -10   0   1   2

pd. (volts)
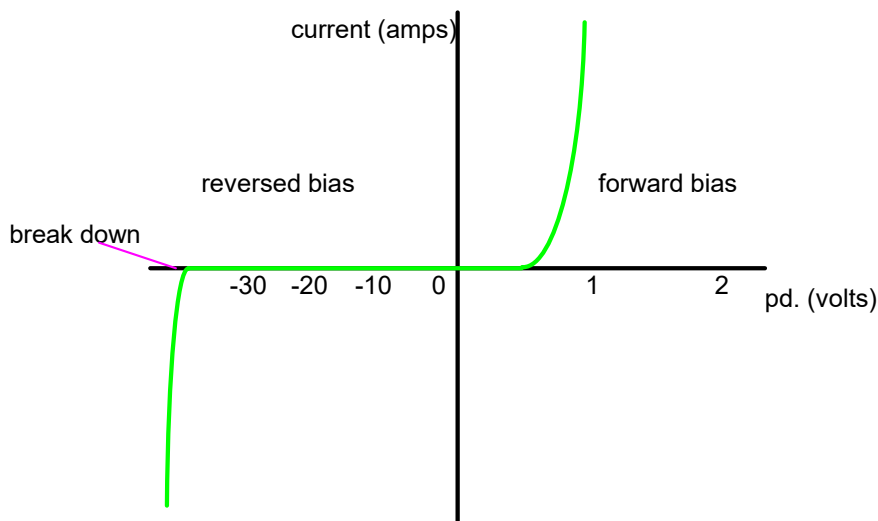
When reversed biased, as the pd. is increased, very little current ($<10^{-9}$A) passes through the diode until the p-n junction breaks down. At this point, a large current can flow, which can destroy the diode.
When forward biased, very little current passes until a pd. of ≈ 0.4V, at which point the current starts to increase. At a pd. of ≈ 0.7V, significant current can pass and this increases exponentially as the pd. is further increased.
The diode formula gives an approximation of the current passing through a forward biased diode

$$I = I_0 \left(\exp\left(\frac{eV}{kT}\right) - 1 \right)$$

Where   I = the forward bias current,
            I_0 = the reverse bias current
            e = the charge on the electron
            V = the forward bias pd.
            k = Boltzmann constant
            T = the absolute temperature.

Types and uses of diodes.
The major use of a standard diode is to control the direction that current passes in a circuit, since a diode only allows current to flow in one direction.
Mains electricity is alternating current (a.c.) and flows in alternating direction, but electronic circuits need direct current (d.c.) which only flow in one direction.
Diodes are used to change a.c. to d.c.

Some diodes are made which emit light when a current passes, Light Emitting Diodes (LEDs) and others are made which are sensitive to light, Photodiodes.