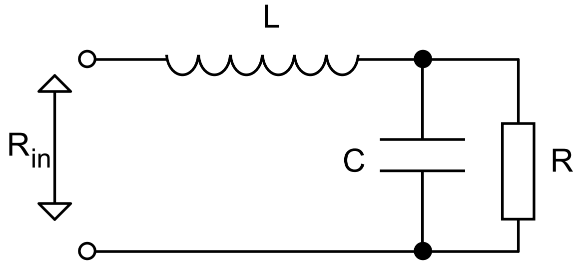
L matching networks.

When the location of an aerial is different to that of a transmitter/receiver, it is necessary to pass the signal between the two with the minimum of loss. This is usually achieved by using a transmission line, often in the form of coaxial cable.
Transmission lines have a characteristic resistance, with 50Ω or 75Ω being common for coaxial cables. When a transmission line is terminated at each end by its characteristic resistance, then stationary waves are not produced and the signal is transferred with minimum loss.
Transmitter power amplifiers and aerials rarely have a resistance that matches the characteristic resistance of a transmission line and so it is necessary to 'match' the transmitter/receiver and the aerial to the transmission line.
The simplest matching circuit is the L match, which has four different arrangements, depending on what is being matched to what.

This article describes the use of a spreadsheet to calculate the component values needed for L-match circuits and also to visualise how changing frequency and component values affects both the impedance and the transfer function.
In most cases, the component values calculated are not preferred values, and the spreadsheet shows the effect of using the nearest preferred values.
The calculations for these circuits are 'interesting' but fortunately can be modelled relatively easily using a spreadsheet by breaking the circuit into small series and parallel sections.

There are four arrangements for the L match circuit and the one to be used depends on the ratio of the input resistance to the output resistance and also on whether the circuit should act as a low pass or high pass filter.
The four circuits are described separately along with their analysis and spreadsheets.

The first L-match circuit is for when **R_{in} is less than R**



$$X_C = R\sqrt{\frac{R_{in}}{R - R_{in}}} \qquad X_L = \frac{R \times R_{in}}{X_C}$$

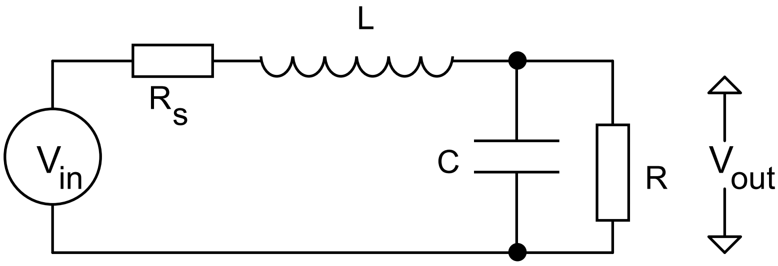
where X_C is the reactance of the capacitor and X_L is the reactance of the inductor.
This circuit will have a resistive input at one frequency. At all other frequencies, the circuit will have an impedance input given by

$$Z_{in} = \frac{R}{\left(1 + R^2/X_C^2\right)} + \left\{ X_L - \frac{R^2}{X_C\left(1 + R^2/X_C^2\right)} \right\}$$

the reactive part of the impedance is in the brackets.

This circuit is modelled in the spreadsheet L-match1.
The frequency at which the circuit has a resistive input is found by looking at the **Phase** column and finding where the phase changes from negative to positive. The input resistance is then found from the **Zin** column.

The spreadsheet L-match1-T shows the transfer function (V_{out}/V_{in}) for this circuit with a source resistance of R_s.



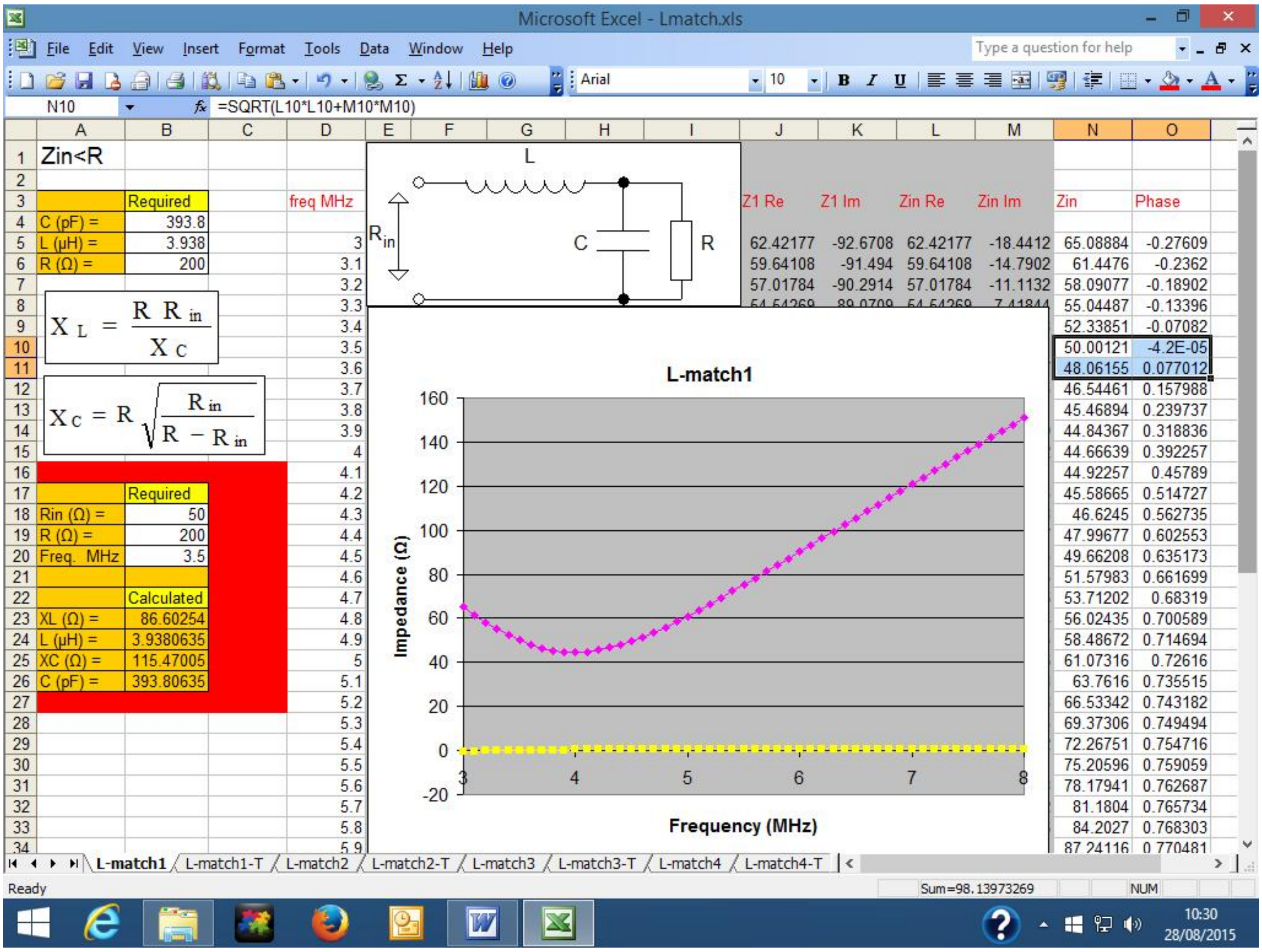
It can be seen from this spreadsheet that the circuit also acts as a low pass filter and so will provide attenuation of harmonics.

Example.

Consider matching a 50Ω voltage source to a load of 200Ω for 3.5MHz.
Substituting these values into the formulae cells on the spreadsheet gives

	Required
Rin (Ω) =	50
R (Ω) =	200
Freq. MHz	3.5
	Calculated
XL (Ω) =	86.60254
L (μH) =	3.9380635
XC (Ω) =	115.47005
C (pF) =	393.80635

X_C = 115.47Ω and X_L = 86.6Ω
If this is to be used at a frequency of 3.5MHz, then
C = 393.8pF and L = 3.938μH
Substituting these values into the spreadsheet Lmatch1 gives

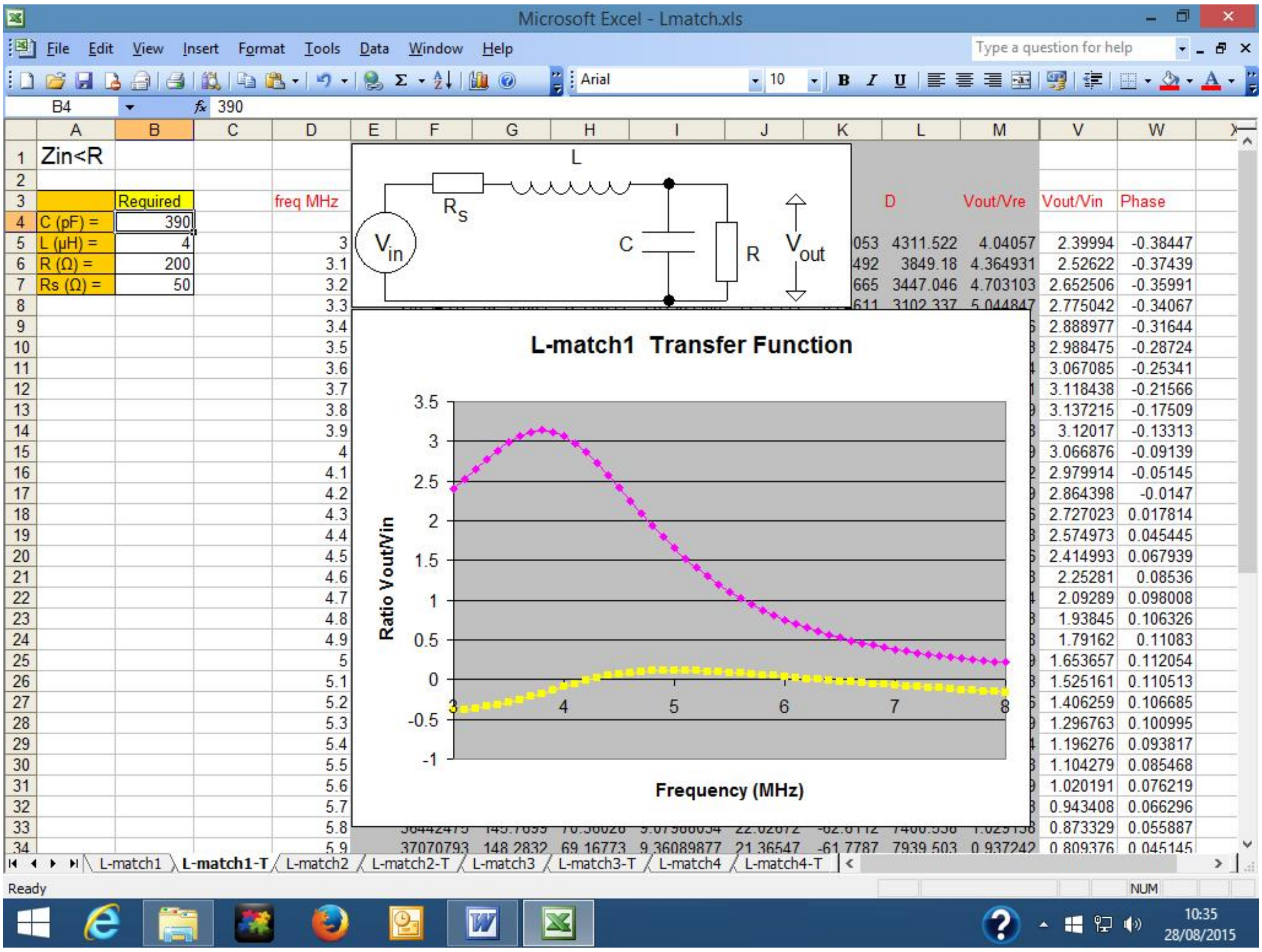


The calculated inductor and capacitor values could be linked directly to the values for the graph but this would prevent investigating how changing the values slightly to account for 'real' component values affects the graph.

The phase column shows that the phase of the input voltage to current changes from negative to positive at 3.5MHz and that the circuit has a resistance of 50Ω at this frequency. This is shown on the right of the picture above.

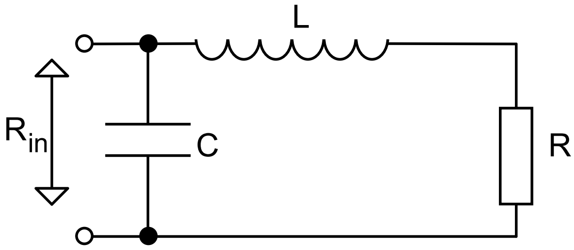
Using real component values, C may well become 390pF and L, 4μH. Using these values in the spreadsheet shows that the effect is negligible.

It is also useful to see the relationship between the input voltage and output voltage. This is shown by inserting the chosen values for L and C into the spreadsheet L-match1-T,



This is useful because it shows that for an input voltage source of 1 volt, approximately 3V would be produced across the 200Ω load.
It is also useful to see that the output voltage decreases with increasing frequency, showing that this circuit is also acting as a low pass filter and helping to reduce harmonic radiation.

The second L-match circuit is for when **R_{in} is greater than R**



$$X_L = \sqrt{R\,R_{in} - R^2} \qquad\qquad X_C = \frac{R\,R_{in}}{X_L}$$

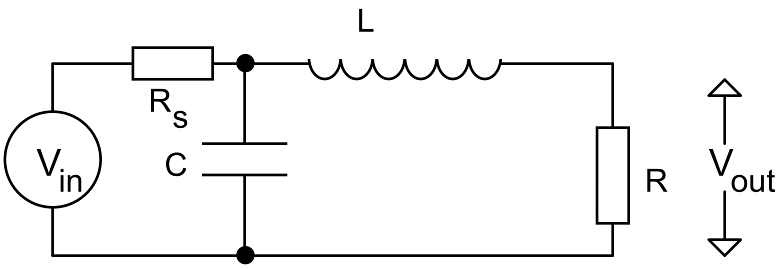
where X_C is the reactance of the capacitor and X_L is the reactance of the inductor.
Again this circuit will have a resistive input at one frequency. At all other frequencies, the circuit will have an impedance input given by

$$Z_{in} = \frac{R}{D} + \left\{ \frac{X_L\left(1 - X_L/X_C\right) - R^2/X_C}{D} \right\} \qquad \text{where } D \equiv \left(1 - X_L/X_C\right)^2 - R^2/X_C^2$$

again, the reactive part of the impedance is in the brackets.

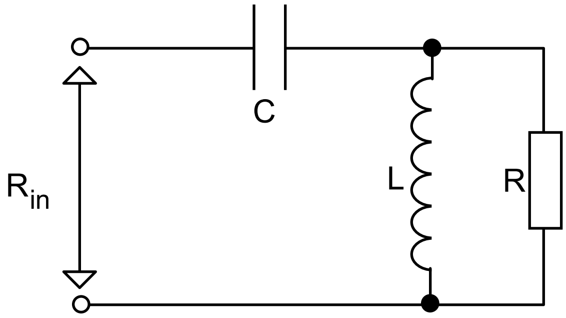
This circuit is modelled in the spreadsheet L-match2.
The frequency at which the circuit has a resistive input is found by looking at the **Phase** column and finding where the phase changes from negative to positive. The input resistance is then found from the **Zin** column.

The spreadsheet L-match2-T shows the transfer function (V_{out}/V_{in}) for this circuit with a source resistance of Rs.



It can be seen from this spreadsheet that the circuit also acts as a low pass filter and so will provide attenuation of harmonics.

The third L-match circuit is again for when **R_{in} is less than R**
This time L and C are swapped around.



$$X_L = R\sqrt{\frac{R_{in}}{R - R_{in}}} \qquad X_C = \frac{R\,R_{in}}{X_L}$$

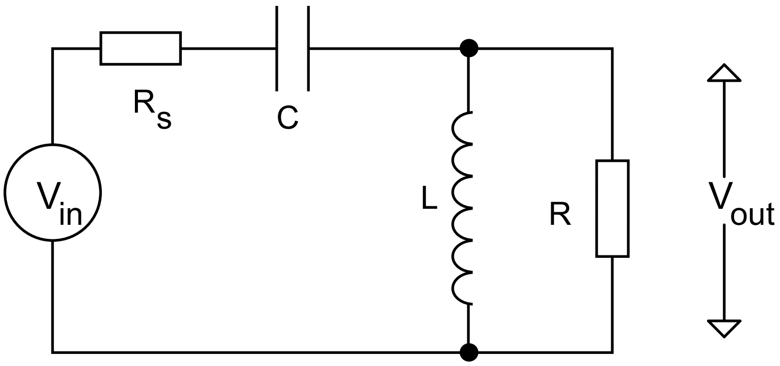
where X_C is the reactance of the capacitor and X_L is the reactance of the inductor.
This circuit will have a resistive input at one frequency. At all other frequencies, the circuit will have an impedance input given by

$$Z_{in} = \frac{R\,X_L^2}{\left(R^2 + X_L^2\right)} + \left\{\frac{R^2 X_L}{R^2 + X_L^2} - X_C\right\}$$

the reactive part of the impedance is in the brackets.

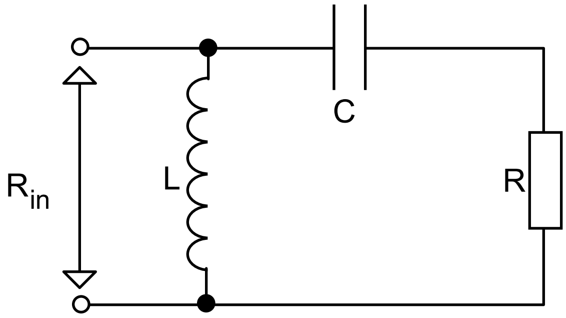
This circuit is modelled in the spreadsheet L-match3.
The frequency at which the circuit has a resistive input is found by looking at the **Phase** column and finding where the phase changes from negative to positive. The input resistance is then found from the **Zin** column.

The spreadsheet L-match3-T shows the transfer function (V_{out}/V_{in}) for this circuit with a source resistance of R_s.



It can be seen from this spreadsheet that the circuit also acts as a high pass filter and so will not attenuate harmonics, which may not be desirable.

The fourth L-match circuit is again for when **R_{in} is greater than R**



$$X_C = \sqrt{R\,R_{in} - R^2} \qquad\qquad X_L = \frac{R\,R_{in}}{X_C}$$

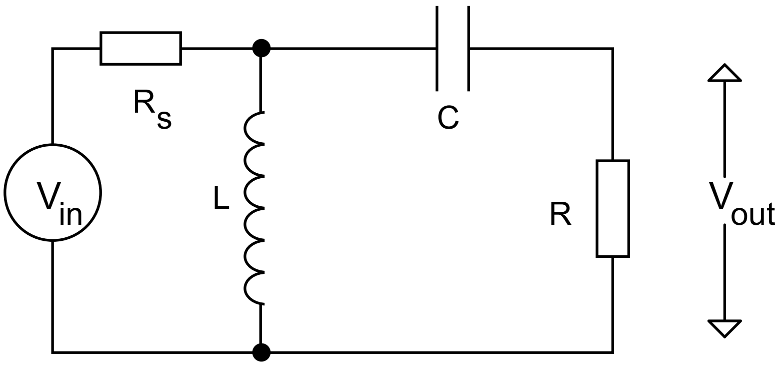
where X_C is the reactance of the capacitor and X_L is the reactance of the inductor.
This circuit will have a resistive input at one frequency. At all other frequencies, the circuit will have an impedance input given by

$$Z_{in} = \left(\frac{R\,X_L^2}{R^2 + (X_L - X_C)^2}\right) + \left\{\frac{R^2 X_L - X_L^2 X_C + X_L X_C^2}{R^2 + (X_L - X_C)^2}\right\}$$

the reactive part of the impedance is in the brackets.

This circuit is modelled in the spreadsheet L-match4.
The frequency at which the circuit has a resistive input is found by looking at the **Phase** column and finding where the phase changes from negative to positive. The input resistance is then found from the **Zin** column.

The spreadsheet L-match4-T shows the transfer function (V_{out}/V_{in}) for this circuit with a source resistance of R_s.



It can be seen from this spreadsheet that the circuit also acts as a high pass filter and so will not attenuate harmonics, which may not be desirable.