## Basic A.C. theory.

An alternating current can be represented by $\mathrm{I}=\mathrm{I}_{0} \sin (\omega \mathrm{t})$
where $\mathrm{I}_{0}$ is the amplitude of the current and $\omega=2 \pi \times$ frequency
For an ideal resistor, the current and voltage are always in phase, so, using Ohm's Law, the alternating voltage across a resistor, $R$, is give

$$
\mathrm{V}=\mathrm{R} \mathrm{I}_{0} \sin (\omega \mathrm{t})
$$

For an ideal inductor, the induced voltage across an inductor when a current, I, is passing is given by Faraday's Law

$$
\mathrm{V}=-\mathrm{L} \frac{\mathrm{dI}}{\mathrm{dt}}
$$

This voltage opposes the voltage causing the current to flow through the inductor, and so for an ideal inductor, the applied voltage must be equal and opposite to this induced voltage.
So the applied voltage is given by

$$
\mathrm{V}=\mathrm{L} \frac{\mathrm{dI}}{\mathrm{dt}}=\mathrm{L} \frac{\mathrm{~d}}{\mathrm{dt}} \mathrm{I}_{0} \sin (\omega \mathrm{t})=\mathrm{L} \omega \mathrm{I}_{0} \cos (\omega \mathrm{t})=\mathrm{V}_{0} \cos (\omega \mathrm{t})
$$

This expression shows that the applied voltage across an inductor leads the current by $90^{\circ}$.
For a capacitor, the voltage across the capacitor is given by the charge, Q , divided by the capacitance, C.

$$
\mathrm{V}=\frac{\mathrm{Q}}{\mathrm{C}}
$$

The charge is given by $\mathrm{Q}=\int \mathrm{I} . \mathrm{dt}$

$$
\text { So } \mathrm{V}=\frac{\mathrm{Q}}{\mathrm{C}}=\frac{1}{\mathrm{C}} \int \mathrm{I} \cdot \mathrm{dt}=\frac{1}{\mathrm{C}} \int \mathrm{I}_{0} \sin (\omega \mathrm{t}) \cdot \mathrm{dt}=-\frac{\mathrm{I}_{0}}{\omega \mathrm{C}} \cos (\omega \mathrm{t})=-\mathrm{V}_{0} \cos (\omega \mathrm{t})
$$

This expression shows that the applied voltage across a capacitor lags the current by $90^{\circ}$.

For any ideal inductor or capacitor, the ratio of the applied voltage to the current passing through is called the reactance, X . This should be compared to Ohm's law where the ratio of voltage to current is called resistance.
For the inductor,

$$
\begin{aligned}
& \mathrm{L} \omega \mathrm{I}_{0} \cos (\omega \mathrm{t})=\mathrm{V}_{0} \cos (\omega \mathrm{t}) \\
& \mathrm{X}_{\mathrm{L}}=\frac{\mathrm{V}_{0}}{\mathrm{I}_{0}}=\omega \mathrm{L}
\end{aligned}
$$

For the capacitor,

$$
\begin{aligned}
& -\frac{\omega \mathrm{I}_{0}}{\mathrm{C}} \cos (\omega \mathrm{t})=-\mathrm{V}_{0} \cos (\omega \mathrm{t}) \\
& \mathrm{X}_{\mathrm{C}}=\frac{\mathrm{V}_{0}}{\mathrm{I}_{0}}=\frac{1}{\omega \mathrm{C}}
\end{aligned}
$$

Since the voltage leads the current in an inductor by $90^{\circ}$, the reactance of an inductor, $\mathrm{X}_{\mathrm{L}}$, is taken as positive, so

$$
\mathrm{X}_{\mathrm{L}}=+\omega \mathrm{L}
$$

Since the voltage lags the current in a capacitor by $90^{\circ}$, the reactance of a capacitor, $\mathrm{X}_{\mathrm{C}}$, is taken as negative, so

$$
X_{C}=-1 / \omega C
$$

A circuit containing both resistive and reactive elements is said to have an Impedance, Z .
Since the reactive elements are out of phase with the resistive element,
the impedance of a circuit is given by $\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}\right)^{2}}$
and the 'phase angle', $\phi$, the angle between the reactive and resistive voltages is given by

$$
\phi=\tan ^{-1}\left(\frac{\mathrm{X}}{\mathrm{R}}\right)
$$

Any impedance ( $Z$ ) can be considered to consist of a purely resistive element (R) and a purely reactive element ( X ), so

$$
\mathrm{Z}=\mathrm{R}+\mathrm{X}
$$

If X is positive, then the reactive component is predominantly inductive. If X is negative, then the reactive component is predominantly capacitive.

